1 Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

$P(TT) = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

$X \sim \text{Geo}(0.25)$

$P(X = 7) = 0.25 \times 0.75^6$

$= 0.0445$

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

$P(X > 9) = q^9$

probability of 9 failures

$= 0.75^9$

$= 0.0751$

2 A summary of 40 values of x gives the following information:

$$\Sigma(x-k) = 520, \qquad \Sigma(x-k)^2 = 9640,$$

where k is a constant.

(a) Given that the mean of these 40 values of x is 34, find the value of k. [2]

$\Sigma(x-k) = 520$

$\Sigma x = 520 + 40k$ ①

$\bar{x} = \dfrac{\Sigma x}{n}$

$34 = \dfrac{\Sigma x}{40}$

$\Sigma x = 1360$ ②

① = ②:

$520 + 40k = 1360$

$40k = 840$

$k = 21$

(b) Find the variance of these 40 values of x. [2]

Variance of $x-k$ = Variance of x

$\text{Variance} = \dfrac{\Sigma(x-k)^2}{n} - \left(\dfrac{\Sigma(x-k)}{n}\right)^2$

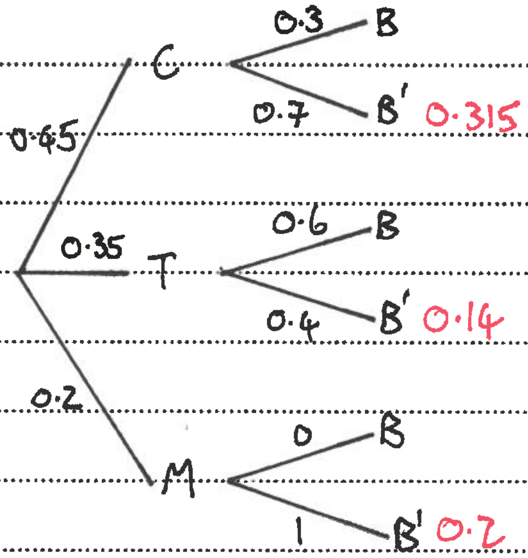
mean of $x-k$

$= \dfrac{9640}{40} - \left(\dfrac{520}{40}\right)^2$

$= 72$

3 For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]



$$P(T \cap B') = P(T|B') \times P(B')$$

$$P(T|B') = \frac{P(T \cap B')}{P(B')}$$

$$= \frac{0.14}{0.315 + 0.14 + 0.2}$$

$$= \frac{0.14}{0.655}$$

$$= \frac{28}{131}$$

4 A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered −1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

	0	1	2	2
-1	-1	0	1	1
0	0	1	2	2
1	1	2	3	3

x	-1	0	1	2	3
$P(X=x)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{3}{12}$	$\frac{2}{12}$

(b) Find $\text{Var}(X)$. [3]

$E(X) = -1\times\frac{1}{12} + 0\times\frac{2}{12} + 1\times\frac{4}{12} + 2\times\frac{3}{12} \times 3\times\frac{2}{12}$

$= \frac{-1}{12} + 0 + \frac{4}{12} + \frac{6}{12} + \frac{6}{12}$

$= \frac{15}{12}$

$Var(X) = (-1)^2\times\frac{1}{12} + 0^2\times\frac{2}{12} + 1^2\times\frac{4}{12} + 2^2\times\frac{3}{12} + 3^2\times\frac{2}{12} - (E(x))^2$

$= \frac{1}{12} + 0 + \frac{4}{12} + \frac{12}{12} + \frac{18}{12} - \left(\frac{15}{12}\right)^2$

$= \frac{35}{12} - \frac{225}{144} = \frac{65}{48}$

5 Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

fixed

_ _ _ _ R _ _ _ _

permutate remaining 8 people into 8 spaces:

$8! = 40\,320$

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

Number of arrangements without any restrictions:

$9! = 362\,880$

Arrangements with R and S together:

_ _ _ _ _ _ _ _

RS

one object

$8! \times 2! = 80\,640$

permutate R and S

$362\,880 - 80\,640 = 282\,240$

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

Not permutating, just picking people for groups.

$${}^{9}C_{5} \times {}^{4}C_{4} = 126$$

pick five people for the first group

pick four from remaining four (= 1)

(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

R and S in back row:

$${}^{2}C_{2} \times {}^{7}C_{3} \times {}^{4}C_{4} = 35$$

pick both R and S for back row

pick 3 more people for the back row

four people for front row

R and S in front row:

$${}^{7}C_{5} \times {}^{2}C_{2} \times {}^{2}C_{2} = 21$$

pick 5 for back row from 7 (not including R and S)

pick R and S for front

pick two more for front

$35 + 21 = 56 \rightarrow$ Probability $= \frac{56}{126} = \frac{4}{9}$

6 The weights, in kg, of 15 rugby players in the Rebels club and 15 soccer players in the Sharks club are shown below.

Rebels	75	78	79	80	82	82	83	84	85	86	89	93	95	99	102
Sharks	66	68	71	72	74	75	75	76	78	83	83	84	85	86	92

(a) Represent the data by drawing a back-to-back stem-and-leaf diagram with Rebels on the left-hand side of the diagram. [4]

Rebels		Sharks
	6	6 8
9 8 5	7	1 2 4 5 5 6 8
9 6 5 4 3 2 2 0	8	3 3 4 5 6
9 5 3	9	2
2	10	

Q2 Q1 Q3

Key: 5|7|1 means 75kg for Rebels and 71kg for Sharks

(b) Find the median and the interquartile range for the Rebels. [3]

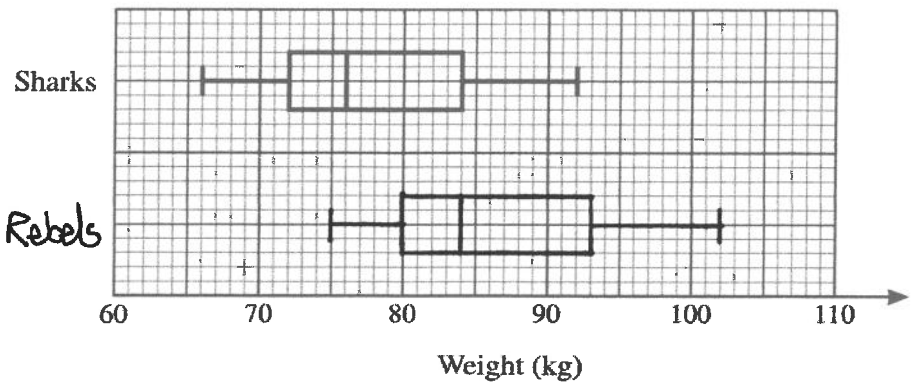
Q_1: $\frac{15+1}{4} = 4^{th}$ Q_2: $\frac{15+1}{2} = 8^{th}$ Q_3: $\frac{3(15+1)}{4} = 12^{th}$

$Q_1 = 80$kg $Q_2 = 84$kg $Q_3 = 93$kg

Median = 84kg IQR = 93 - 80

= 13kg

A box-and-whisker plot for the Sharks is shown below.



(c) On the same diagram, draw a box-and-whisker plot for the Rebels. [2]

(d) Make one comparison between the weights of the players in the Rebels club and the weights of the players in the Sharks club. [1]

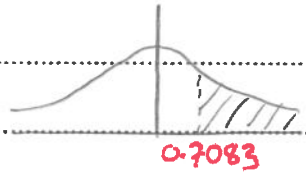
On average, Rebels players are heavier than Sharks players : Rebels median = 84kg
Sharks median = 76kg

7 The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) (i) On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

$P(K > 142) = P\left(Z > \frac{142-125}{24}\right)$

$= P(Z > 0.7083)$



$= 1 - \Phi(0.7083)$

$= 1 - 0.7604$

$= 0.2396$

number of days $= 0.2396 \times 365$

$= 87.454$

$\approx$ 87 days

(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

$T \sim B(10, 0.2396)$

$P(T < 2) = P(T=0) + P(T=1)$

$= {}^{10}C_0 \times 0.2396^0 \times 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$

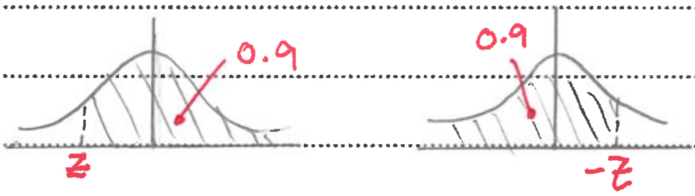
$= 0.268$

(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

$P(K > t) = 0.9$

$P\left(Z > \frac{t - 125}{24}\right) = 0.9$



$\underline{0.9} = \Phi(1.282)$

critical value

$\therefore z = -1.282$

$\frac{t - 125}{24} = -1.282$

$t - 125 = -30.768$

$t = 94.232$

$t \simeq \underline{94.2 \text{ days}}$