1 Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $P(A \cap B)$. [2]

Red

Blue	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(A \cap B)$:

$$P(A \cap B) = \frac{5}{36}$$

(b) Hence determine whether or not the events A and B are independent. [2]

If independent, $P(A) \times P(B) = P(A \cap B)$

$$P(A) = \frac{12}{36} \qquad P(B) = \frac{10}{36}$$

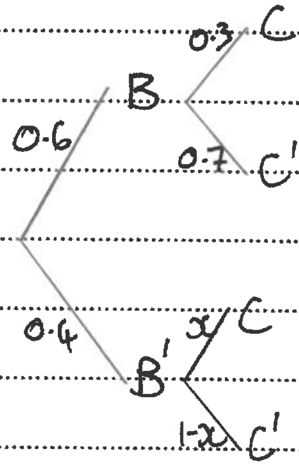
$$P(A) \times P(B) = \frac{12}{36} \times \frac{10}{36}$$

$$= \frac{5}{54}$$

$\frac{5}{54} \neq \frac{5}{36}$ so not independent.

2 The probability that a student at a large music college plays in the band is 0.6. For a student who plays in the band, the probability that she also sings in the choir is 0.3. For a student who does not play in the band, the probability that she sings in the choir is x. The probability that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. [3]



P(not in choir) = $0.6 \times 0.7 + 0.4(1-x)$

$= 0.42 + 0.4 - 0.4x$

$0.58 = 0.82 - 0.4x$

$0.4x = 0.24$

$x = 0.6$

Two students from the college are chosen at random.

(b) Find the probability that both students play in the band and both sing in the choir. [2]

$P(BC) \times P(BC)$

$= 0.6 \times 0.3 \times 0.6 \times 0.3$

$= 0.18 \times 0.18$

$= 0.0324$

3 Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

$X \sim Geo(0.25)$

$P(X > 6) = q^6$

← probability of 6 failures

$= 0.75^6$

$= \underline{0.178}$

(b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

$X \sim B(10, 0.25)$

$P(X \geq 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - ({}^{10}C_0 \times 0.25^0 \times 0.75^{10} + {}^{10}C_1 \times 0.25^1 \times 0.75^9 + {}^{10}C_2 \times 0.25^2 \times 0.75^8)$

$= \underline{0.474}$

4 The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

y	1	2	3	4
$P(Y=y)$	$7/16$	$5/16$	$3/16$	$1/16$

(b) Find the probability that $Y = 2$ given that Y is even. [2]

$P(2 \cap E) = P(2|E) \times P(E)$

$P(2|E) = \dfrac{P(2 \cap E)}{P(E)}$

$P(2 \cap E) = \dfrac{5}{16}$

$P(E) = \frac{5}{16} + \frac{1}{16}$

$= \dfrac{6}{16}$

$P(2|E) = \dfrac{\frac{5}{16}}{\frac{6}{16}}$

$= \dfrac{5}{6}$

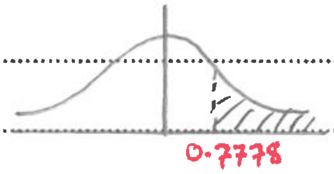
[Turn over

5 The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

$P(T > 4.2) = P\left(Z > \frac{4.2 - 3.5}{0.9}\right)$

$= P(Z > 0.7778)$



$= 1 - \Phi(0.778)$

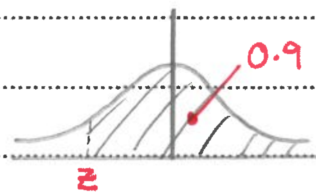
$= 1 - 0.7818$

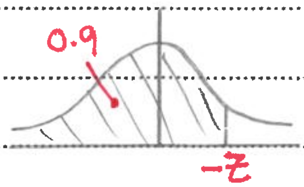
$= 0.2182$

(b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t. [3]

$P(T > t) = 0.9$

$P\left(Z > \frac{t - 3.5}{0.9}\right) = 0.9$





$0.9 = \Phi(1.282)$

critical value

$\therefore\ z = -1.282$

$\dfrac{t - 3.5}{0.9} = -1.282$

$t - 3.5 = -1.1538$

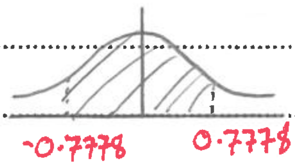
$t = \underline{2.3462}$

(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

$P(2.8 < T < 4.2) = P\left(\dfrac{2.8-3.5}{0.9} < Z < \dfrac{4.2-3.5}{0.9}\right)$

$= P(-0.7778 < Z < 0.7778)$

$= P(Z < 0.7778) - P(Z < -0.7778)$



$= \Phi(0.778) - (1 - \Phi(0.778))$

$= 0.7818 - (1 - 0.7818)$

$= \underline{0.5636}$

Number of days $= 0.5636 \times 365$

$= 205.714$

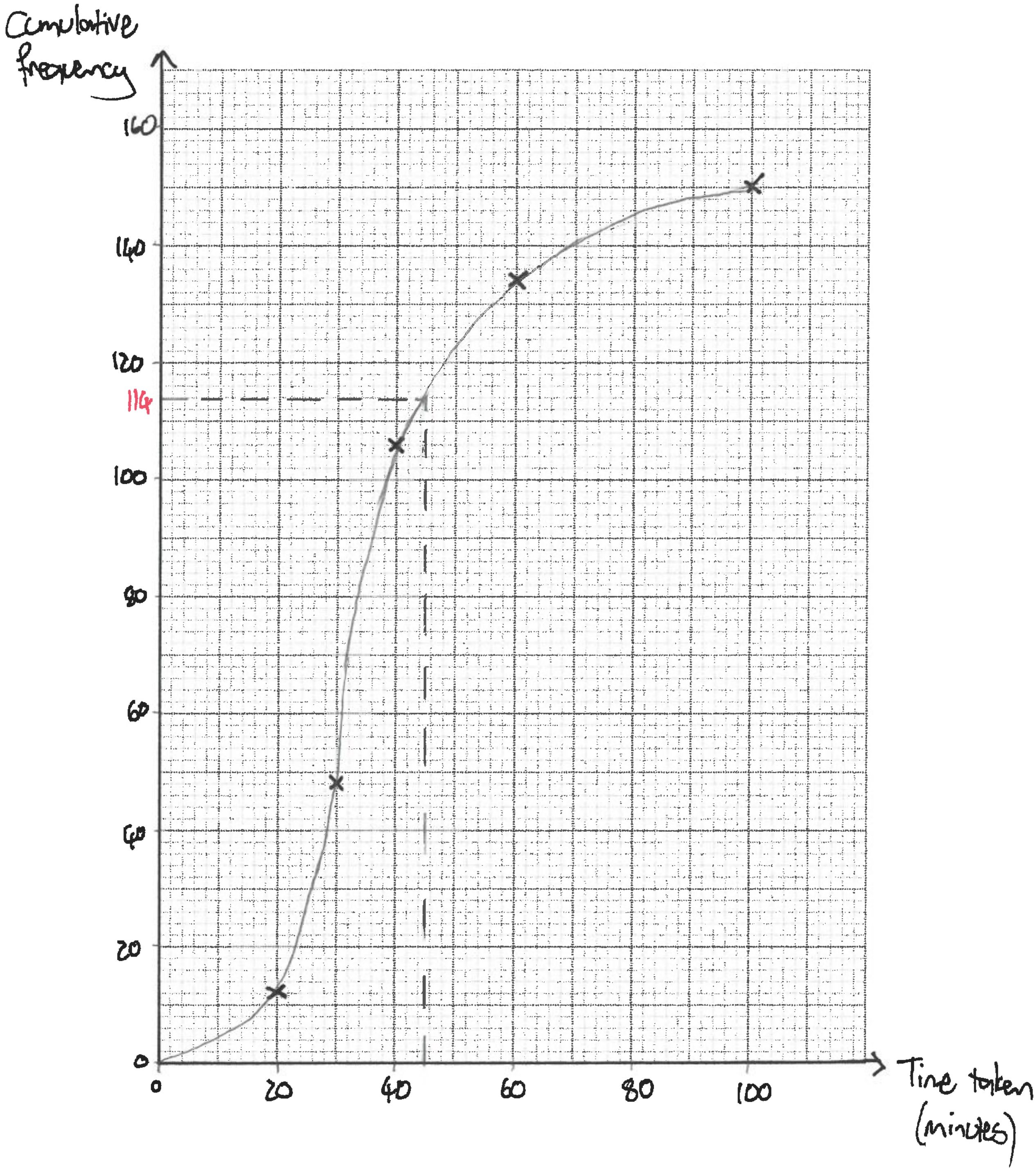
$\approx \underline{206 \text{ days}}$ *

* I disagree with the mark scheme's 205. As this is a probability, 206 is more likely than 205.

6 The times, t minutes, taken by 150 students to complete a particular challenge are summarised in the following cumulative frequency table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 40$	$t \leqslant 60$	$t \leqslant 100$
Cumulative frequency	12	48	106	134	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) 24% of the students take k minutes or longer to complete the challenge. Use your graph to estimate the value of k. [2]

24% take k mins or longer, so 76% take less than k mins.

$0.76 \times 150 = 114$

$k = 45$ mins

(c) Calculate estimates of the mean and the standard deviation of the time taken to complete the challenge. [6]

Mid-point (t)	Frequency (f)	$f \times t$
10	12	120
25	36	900
35	58	2030
50	28	1400
80	16	1280
	$\Sigma f = 150$	$\Sigma ft = 5730$

$$\bar{t} = \frac{5730}{150} = 38.2$$

$$Var = \frac{10^2 \times 12 + 25^2 \times 36 + 35^2 \times 58 + 50^2 \times 28 + 80^2 \times 16}{150} - 38.2^2$$

$= 321.76$

$\sigma = \sqrt{Var}$

$= 17.9$

7 (a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

SHOPPKEEER

E E E

one object

$$\frac{8!}{2!} = 20160$$

2 Ps

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

Ps next to each other:

P P

one object

$$\frac{9!}{3!} = 60480$$

3 Es

No restrictions:

$$\frac{10!}{3! \times 2!} = 302400$$

3 Es

2 Ps

$302400 - 60480 = 241920$

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

E _ _ _ _ _ _ _ _ E

fixed fixed

$$\frac{8!}{2!} = 20\,160$$

2 Ps

$$\text{Probability} = \frac{20160}{302\,400}$$

from part (b)

$$= \frac{1}{15}$$

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

1P, No Es: P _ _ _ $\quad {}^5C_3 = 10$

pick 3 from S, H, O, K, R

1P, 1E : P E _ _ $\quad {}^5C_2 = 10$

1P, 2Es: P E E _ $\quad {}^5C_1 = 5$

1P, 3Es: P E E E $\quad {}^5C_0 = 1$

$$10 + 10 + 5 + 1 = 26$$

Note that we want different selections, so Ps and Es are indistinguishable, so no ${}^2C_1$ for Ps.