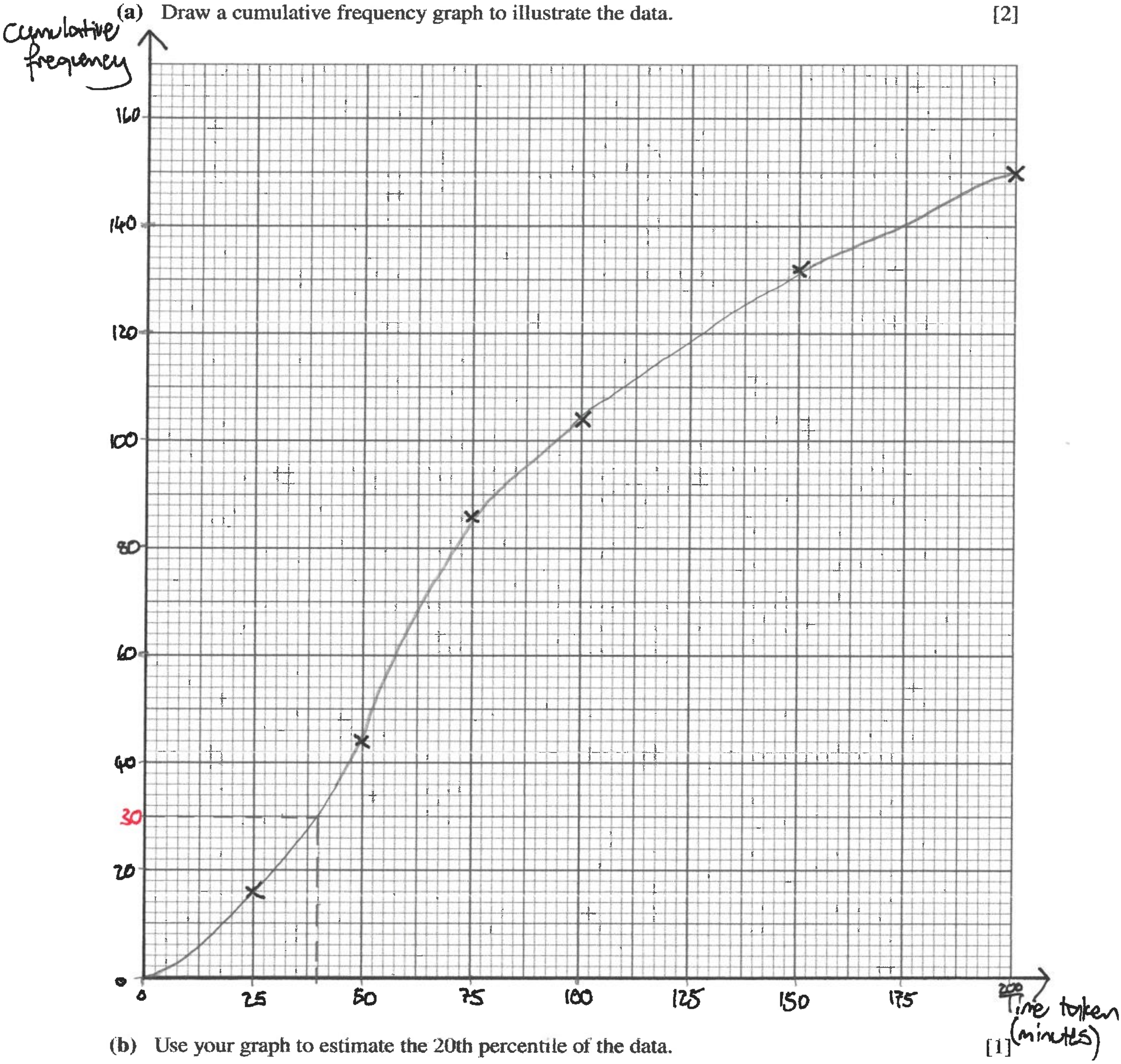
1 The time taken, t minutes, to complete a puzzle was recorded for each of 150 students. These times are summarised in the table.

Time taken (t minutes)	$t \leqslant 25$	$t \leqslant 50$	$t \leqslant 75$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$
Cumulative frequency	16	44	86	104	132	150

(a) Draw a cumulative frequency graph to illustrate the data. [2]



(b) Use your graph to estimate the 20th percentile of the data. [1]

$0.2 \times 150 = 30$

20th percentile = 40

2 Twenty children were asked to estimate the height of a particular tree. Their estimates, in metres, were as follows.

4.1	4.2	4.4	4.5	4.6	4.8	5.0	5.2	5.3	5.4
5.5	5.8	6.0	6.2	6.3	6.4	6.6	6.8	6.9	19.4

(a) Find the mean of the estimated heights. [1]

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{123.4}{20}$$

$$= \underline{6.17\text{ m}}$$

(b) Find the median of the estimated heights. [1]

$$\frac{5.4 + 5.5}{2} = \underline{5.45\text{ m}}$$

(c) Give a reason why the median is likely to be more suitable than the mean as a measure of the central tendency for this information. [1]

The data contains an outlier: 19.4, which affects the mean, but not the median.

3 The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

$x = -2$: $k(-2)^2 = 4k$

$x = 1$: $k(1)^2 = k$

$x = 2$: $k(2)^2 = 4k$

$x = 3$: $k(3)^2 = 9k$

$\Sigma P = 1$: $4k + k + 4k + 9k = 1$

$18k = 1$

$k = \frac{1}{18}$

x	-2	1	2	3
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

(b) Find $E(X)$ and $Var(X)$. [3]

$E(X) = -2 \times \frac{4}{18} + 1 \times \frac{1}{18} + 2 \times \frac{4}{18} + 3 \times \frac{9}{18}$

$= \frac{-8}{18} + \frac{1}{18} + \frac{8}{18} + \frac{27}{18}$

$= \frac{28}{18} = \underline{\frac{14}{9}}$

$Var(X) = (-2)^2 \times \frac{4}{18} + 1^2 \times \frac{1}{18} + 2^2 \times \frac{4}{18} + 3^2 \times \frac{9}{18} - (E(X))^2$

$= \frac{16}{18} + \frac{1}{18} + \frac{16}{18} + \frac{81}{18} - \left(\frac{14}{9}\right)^2$

$= \frac{114}{18} - \frac{196}{81} = \underline{\frac{317}{81}}$

4 Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

$X \sim Geo\left(\frac{1}{6}\right)$

$P(X=8) = \frac{1}{6} \times \left(\frac{5}{6}\right)^7$

$= \underline{0.0465}$

(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

$P(X \leqslant 5) = 1 - q^5$

probability of 5 failures

$= 1 - \left(\frac{5}{6}\right)^5$

$= \underline{0.598}$

Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

A	B
1	1
1	2
2	1

$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ $\quad P(\text{less than } 4) = 3 \times \frac{1}{36}$

$= \underline{\frac{1}{12}}$

$Y \sim B\left(10, \frac{1}{12}\right)$

$P(Y \geqslant 3) = 1 - (P(0) + P(1) + P(2))$

$= 1 - \left({}^{10}C_0 \times \left(\frac{1}{12}\right)^0 \times \left(\frac{11}{12}\right)^{10} + {}^{10}C_1 \times \left(\frac{1}{12}\right)^1 \times \left(\frac{11}{12}\right)^9 + {}^{10}C_2 \times \left(\frac{1}{12}\right)^2 \times \left(\frac{11}{12}\right)^8\right)$

$= \underline{0.0445}$

5 Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

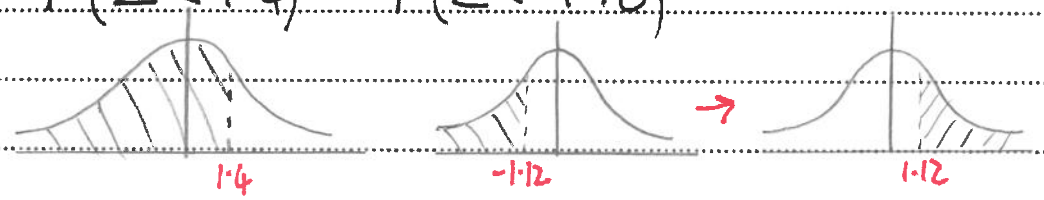
(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

$P(142 < W < 205)$

$P\left(\frac{142-170}{25} < Z < \frac{205-170}{25}\right)$

$P(-1.12 < Z < 1.4)$

$= P(Z < 1.4) - P(Z < -1.12)$



$= \Phi(1.4) - (1 - \Phi(1.12))$

$= 0.9192 - (1 - 0.8686)$

$= \underline{0.7878}$

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

$P(W > 205) = P(Z > 1{\cdot}4)$ (part (a))

$= 1 - \Phi(1{\cdot}4)$

$= 1 - 0{\cdot}9192$

$= 0{\cdot}0808$

large apples: $0{\cdot}0808 \times 20000 = 1616$

$1616 \times 0.30 = \$484.80$

normal apples: $0{\cdot}7878 \times 20000 = 15756$

$15756 \times 0.24 = \$3781.44$

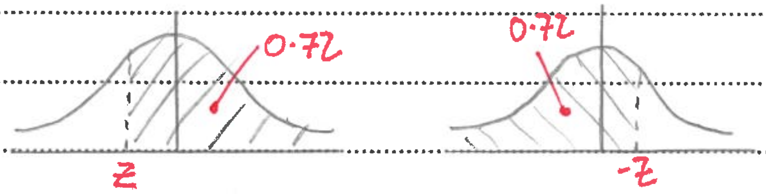
Total: $484.80 + 3781.44 = \underline{\underline{\$4266.24}}$

Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

$P(W > w) = 0{\cdot}72$

$P\left(Z > \frac{w - 182}{20}\right) = 0{\cdot}72$



$0{\cdot}72 = \Phi(0{\cdot}583)$

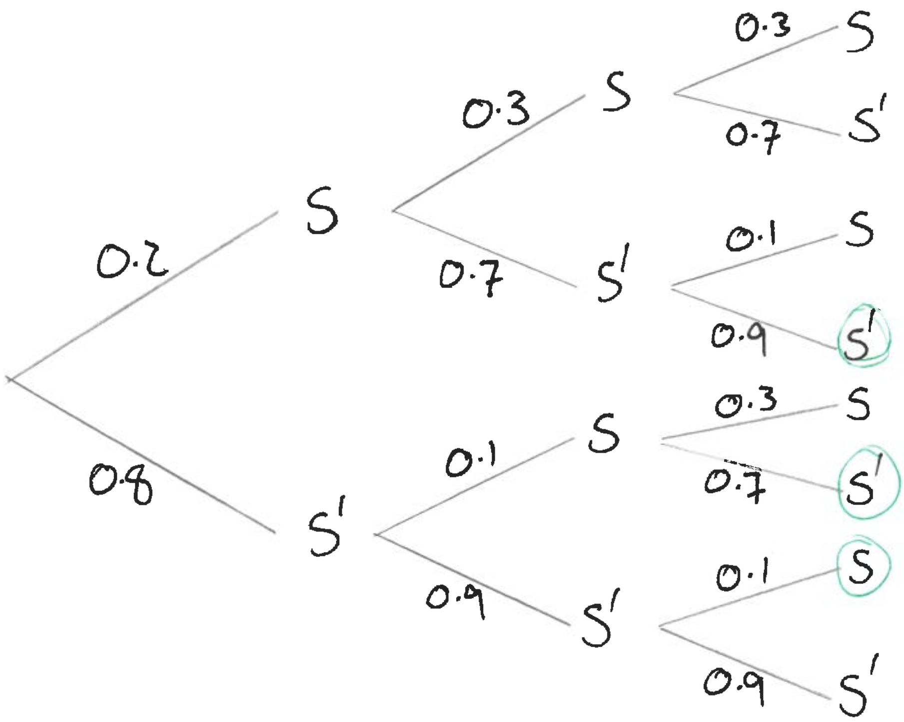
$\therefore z = -0{\cdot}583$

$\frac{w - 182}{20} = -0{\cdot}583 \rightarrow w - 182 = -11{\cdot}66$

$w = \underline{170{\cdot}34g}$

6 Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]



(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

$P(X)$ $P(Y)$

$P(X)$: see ◯ on diagram

$P(X) = 0.2\times0.7\times0.9 + 0.8\times0.1\times0.7 + 0.8\times0.9\times0.1$

$= 0.126 + 0.056 + 0.072$

$= \underline{0.254}$

$P(Y) = 1 - P(S'S'S')$

$= 1 - 0.8\times0.9\times0.9$

$= 1 - 0.648$

$= \underline{0.352}$

$P(X|Y) = \dfrac{P(X\cap Y)}{P(Y)}$

$= \dfrac{0.254}{0.352}$

$= \underline{0.722}$

$P(X\cap Y) = P(X|Y)\times P(Y)$

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

$P(SSSS'S'S') = 0.2\times0.3\times0.3\times0.7\times0.9\times0.9 = 0.010206$

$P(S'S'S'SSS) = 0.8\times0.9\times0.9\times0.1\times0.3\times0.3 = 0.005832$

$P(SSSS'S'S') + P(S'S'S'SSS) = \underline{0.016038}$

7 A group of 15 friends visit an adventure park. The group consists of four families.

- Mr and Mrs Kenny and their four children
- Mr and Mrs Lizo and their three children
- Mrs Martin and her child
- Mr and Mrs Nantes

The group travel to the park in three cars, one containing 6 people, one containing 5 people and one containing 4 people. The cars are driven by Mr Lizo, Mrs Martin and Mr Nantes respectively.

(a) In how many different ways can the remaining 12 members of the group be divided between the three cars? [3]

ie. one group of 5, one of 4 and one of 3.

$${}^{12}C_5 \times {}^{7}C_4 \times {}^{3}C_3 = 27720$$

pick 5 people from 12 for first car.

pick 4 from the remaining 7

pick 3 from 3

The group enter the park by walking through a gate one at a time.

(b) In how many different orders can the 15 friends go through the gate if Mr Lizo goes first and each family stays together? [3]

Mr Lizo fixed

4!

Kenny family 6!

Martins 2!

Nantes 2!

in any order, so × 3!

$$4! \times 6! \times 2! \times 2! \times 3! = 414720$$

In the park, the group enter a competition which requires a team of 4 adults and 3 children.

(c) In how many ways can the team be chosen from the group of 15 so that the 3 children are all from different families? [2]

$$ {}^{7}C_{4} \times {}^{4}C_{1} \times {}^{3}C_{1} \times {}^{1}C_{1} = \underline{420} $$

pick 4 adults from 7

1 child from Kenny family

1 from Lizos

1 from Martins

(d) In how many ways can the team be chosen so that at least one of Mr Kenny or Mr Lizo is included? [3]

Only one of Mr Kenny or Mr Lizo:

$$ {}^{2}C_{1} \times {}^{5}C_{3} \times {}^{8}C_{3} = 1120 $$

pick one from Mr Kenny & Mr Lizo

3 more adults from

pick 3 children from all 8.

Both Mr Kenny and Mr Lizo:

$$ {}^{2}C_{2} \times {}^{5}C_{2} \times {}^{8}C_{3} = 560 $$

both Mr Kenny & Mr Lizo

2 more adults

children

$$ 1120 + 560 = \underline{1680} $$