1 For n values of the variable x, it is given that

$$\Sigma(x - 200) = 446 \quad \text{and} \quad \Sigma x = 6846. \quad ①$$

Find the value of n. [3]

$$\Sigma(x - 200) = 446$$
$$\Sigma x = 446 + 200n \quad ②$$

$$① = ②$$

$$446 + 200n = 6846$$
$$200n = 6400$$
$$n = 32$$

2 A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

x	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(b) Find $E(X)$ and $Var(X)$. [3]

$$E(X) = 2\times\frac{1}{36} + 3\times\frac{4}{36} + 4\times\frac{10}{36} + 5\times\frac{12}{36} + 6\times\frac{9}{36}$$

$$= \frac{2}{36} + \frac{12}{36} + \frac{40}{36} + \frac{60}{36} + \frac{54}{36}$$

$$= \underline{\frac{14}{3}}$$

$$Var(X) = 2^2\times\frac{1}{36} + 3^2\times\frac{4}{36} + 4^2\times\frac{10}{36} + 5^2\times\frac{12}{36} + 6^2\times\frac{9}{36} - (E(X))^2$$

$$= \frac{4}{36} + \frac{36}{36} + \frac{160}{36} + \frac{300}{36} + \frac{324}{36} - \left(\frac{14}{3}\right)^2$$

$$= \frac{206}{9} - \frac{196}{9} = \underline{\frac{10}{9}}$$

3 The back-to-back stem-and-leaf diagram shows the diameters, in cm, of 19 cylindrical pipes produced by each of two companies, A and B.

Company A							Company B				
					4	33	1	2	8		
	9	Q1 8	3	2	0	34	1	6	8	9	9
8	7	Q2 5	4	1	1	35	1	2	2	3	
		9	Q3 6	5	2	36	5	6			
			4	3	1	37	0	3	4		
						38	2	8			

Key: 1 | 35 | 3 means the pipe diameter from company A is 0.351 cm and from company B is 0.353 cm.

(a) Find the median and interquartile range of the pipes produced by company A. [3]

Q_1: $\frac{19+1}{4} = 5^{th}$ Q_2: $\frac{19+1}{2} = 10^{th}$ Q_3: $\frac{3(19+1)}{4} = 15^{th}$

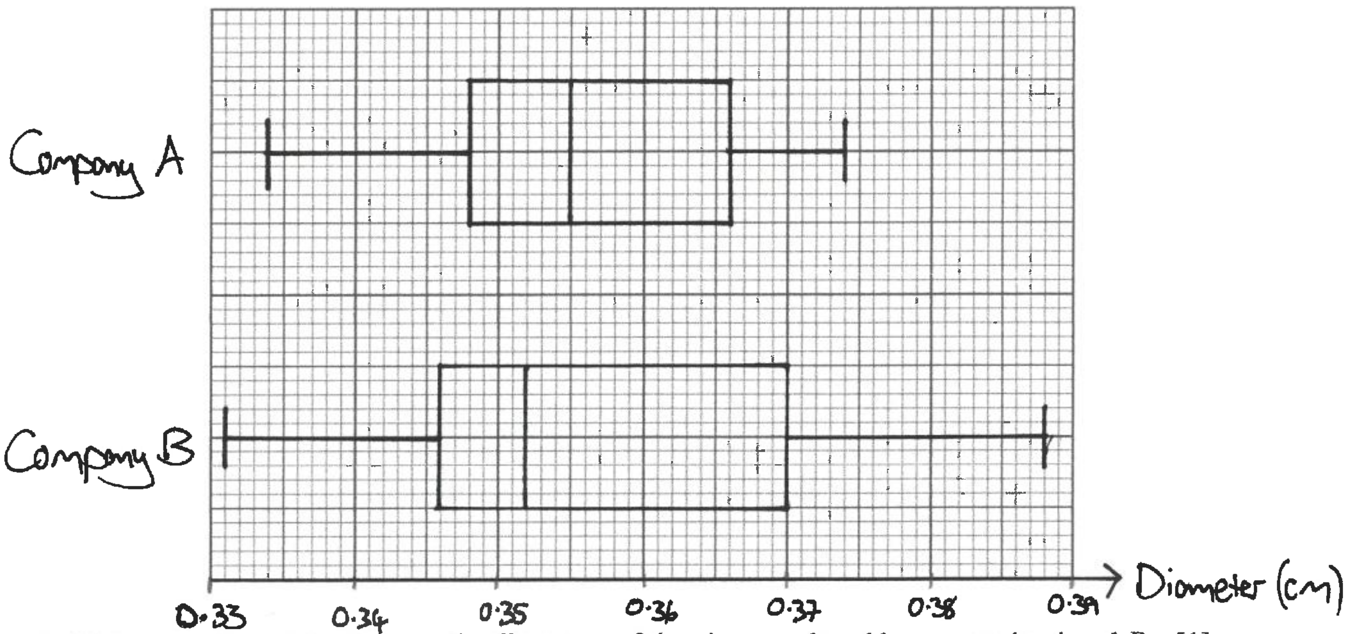
$Q_1 = 0.348$ $Q_2 = 0.355$ $Q_3 = 0.366$

Median $= 0.355$ cm IQR $= 0.366 - 0.348$

$= 0.018$

It is given that for the pipes produced by company B the lower quartile, median and upper quartile are 0.346 cm, 0.352 cm and 0.370 cm respectively.

(b) Draw box-and-whisker plots for companies A and B on the grid below. [3]



(c) Make one comparison between the diameters of the pipes produced by companies A and B. [1]

On average, the pipes produced by company A have a larger diameter than those from company B.

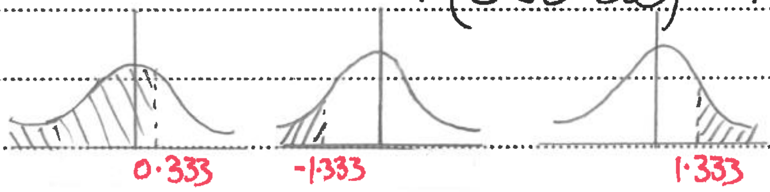
4 The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

$P(1.98 < W < 2.03) = P\left(\frac{1.98-2.02}{0.03} < Z < \frac{2.03-2.02}{0.03}\right)$

$= P(-1.333 < Z < 0.333)$

$= P(Z < 0.333) - P(Z < -1.333)$



$= \Phi(0.333) - (1 - \Phi(1.333))$

$= 0.6304 - (1 - 0.9087)$

$= 0.5391$

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

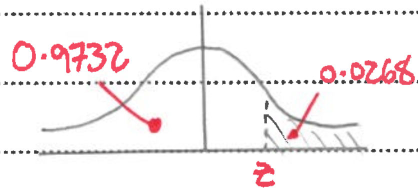
(b) Find the value of σ. [4]

$P(W > 2.6) = \frac{134}{5000}$

$= 0.0268$

$P(W > 2.6) = 0.0268$

$P\left(Z > \frac{2.6 - 2.55}{\sigma}\right) = 0.0268$



$0.9732 = \Phi(1.93)$ from table

$\therefore z = 1.93$

$\frac{2.6 - 2.55}{\sigma} = 1.93$

$0.05 = 1.93\sigma$

$\sigma = \underline{\underline{0.0259}}$

5 In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

$M \sim B(12, 0.72)$

one instrument + two or more instruments

$P(M > 9) = P(10) + P(11) + P(12)$

$= {}^{12}C_{10} \times 0.72^{10} \times 0.28^{2} + {}^{12}C_{11} \times 0.72^{11} \times 0.28^{1} + {}^{12}C_{12} \times 0.72^{12} \times 0.28^{0}$

$= 0.304$

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

$X \sim B(90, 0.52)$ ← exactly one instrument

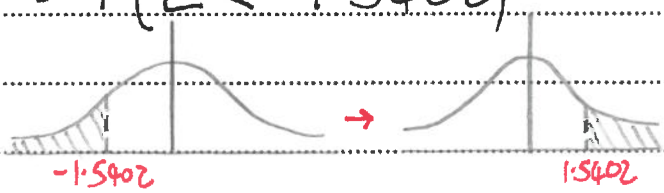
$\mu = 90 \times 0.52$ $\qquad$ $\sigma^2 = 46.8(0.48)$

$= 46.8$ $\qquad$ $= 22.464$

$P(X < 40) \rightarrow P(X < 39.5)$ (continuity correction)

$P\left(Z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$

$= P(Z < -1.5402)$



$= 1 - \Phi(1.540)$

$= 1 - 0.9382$

$= 0.0618$

6 (a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

CCROODILE

$$\frac{9!}{2! \times 2!} = \underline{90\,720}$$

2Cs 2Os

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

With Os together:

C _ _ _ _ _ _ C

fixed (OO) fixed

one object

$6! = 720$

No restrictions (but with Cs at ends):

C _ _ _ _ _ _ _ C

fixed fixed

$$\frac{7!}{2!} = 2520$$

2 Os

$2520 - 720 = \underline{1800}$

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

1C, No Os C _ _ _ $^5C_3 = 10$

pick 3 letters from R, D, I, L, E

2Cs, No Os C C _ _ $^5C_2 = 10$

2Cs, 1O C C O _ $^5C_1 = 5$

1O O _ _ _ $^5C_3 = 10$

2Os O O _ _ $^5C_2 = 10$

$10+10+5+10+10+5 = \underline{50}$

2Os, 1C O O C _ $^5C_1 = 5$

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

difficult!

1C with 2Os: C O O C _ _ _ _ _

pick 2 from RDILE $^5C_2 \times {}^3C_3 = \underline{10}$

1C with O, one without C O _ C _ _ O _ _

$^5C_1 \times {}^4C_2 \times {}^2C_2 = \underline{30}$

each C with O: C O _ C O _ _ _ _

$^5C_1 \times {}^4C_1 \times {}^3C_3 = 20$

BUT: Imagine we picked R for the first group and D for the second:

C O R C O D _ _ _

We could also have picked D in the first group and R in the second:

C O D C O R _ _ _

Ending up with the same groups!

So we have to account for the fact that the first two groups can give identical results by dividing by 2! (Like if we had two identical letters in a word).

$$\rightarrow \frac{20}{2!} = \underline{10}$$

Two Os without C:

C ___ ___ C ___ ___ O O ___

$${}^5C_2 \times {}^3C_2 \times {}^1C_1 = 30$$

Once again, the first two groups can give identical results, so:

$$\frac{30}{2!} = \underline{15}$$

$$\rightarrow 10 + 30 + 10 + 15 = \underline{65}$$

7 Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

$P(RRR) = \frac{^3C_3}{^{12}C_3} = \frac{1}{220}$

$P(OOO) = \frac{^4C_3}{^{12}C_3} = \frac{1}{55}$

$P(YYY) = \frac{^5C_3}{^{12}C_3} = \frac{1}{22}$

$P(RRR) + P(OOO) + P(YYY) = \frac{3}{44}$

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

$P(YYY) = \frac{1}{22}$

$P(\text{all same}) = \frac{3}{44}$

$P(YYY \cap \text{all same}) = P(YYY \mid \text{all same}) \times P(\text{all same})$

$P(YYY \mid \text{all same}) = \frac{P(YYY \cap \text{all same})}{P(\text{all same})}$

$= \frac{1/22}{3/44} = \frac{2}{3}$

(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

1 orange 2 others

$P(\text{one orange}) = \frac{{}^4C_1 \times {}^8C_2}{{}^{12}C_3} = \frac{28}{55}$

$P(\text{two orange}) = \frac{{}^4C_2 \times {}^8C_1}{{}^{12}C_3} = \frac{12}{55}$

$P(\text{three orange}) = \frac{{}^4C_3}{{}^{12}C_3} = \frac{1}{55}$

$P(\text{at least one orange}) = \frac{28}{55} + \frac{12}{55} + \frac{1}{55}$

$= \frac{41}{55}$