1 **(a)** Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

DDEEECIV

EEE DD

one object one object

$5! = 120$

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

With 3 Es together:

EEE

one object

$\frac{6!}{2!} = 360$

2 Ds

No restrictions:

$\frac{8!}{3! \times 2!} = 3360$

3 Es 2Ds

$3360 - 360 = 3000$

2 There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

$^{2}C_1 \times {}^{12}C_4 = \underline{990}$

pick one of Mr/Mrs Lan

pick 4 members from the remaining 12 (excluding Mr & Mrs Lan)

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

3 Women (inc. Mrs Lan), 2 Men:

Mrs Lan _W_ _W_ _M_ _M_

$^{7}C_2 \times {}^{6}C_2 = 315$

pick 2 more women from 7

pick 2 men from 6

4 Women (inc. Mrs Lan), 1 Man:

Mrs Lan _W_ _W_ _W_ _M_

$^{7}C_3 \times {}^{6}C_1 = 210$

5 Women (inc. Mrs Lan):

Mrs Lan _W_ _W_ _W_ _W_

$^{7}C_4 = 35$

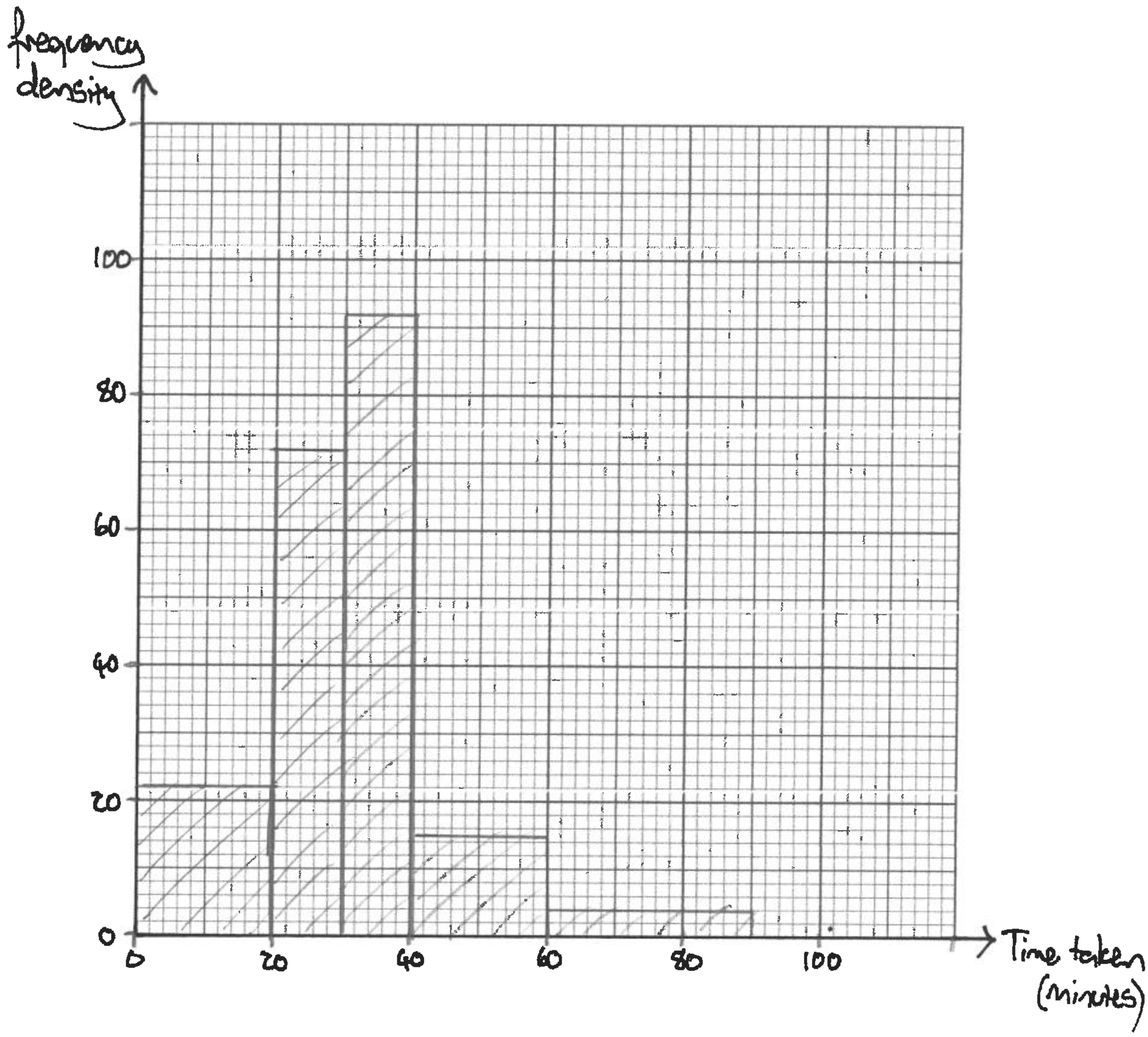
$315 + 210 + 35 = \underline{560}$

3 The times taken to travel to college by 2500 students are summarised in the table.

Time taken (t minutes)	$0 \leqslant t < 20$	$20 \leqslant t < 30$	$30 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 90$
Frequency	440	720	920	300	120

(a) Draw a histogram to represent this information. [4]

Class width	20	10	10	20	30
f.d.	22	72	92	15	4



From the data, the estimate of the mean value of t is 31.44.

(b) Calculate an estimate of the standard deviation of the times taken to travel to college. [3]

Mid-points: 10, 25, 35, 50, 75

$$Var = \frac{10^2 \times 440 + 25^2 \times 720 + 35^2 \times 920 + 50^2 \times 300 + 75^2 \times 120}{2500} - 31.44^2$$

$= 229.9264$

$\sigma = \sqrt{229.9264}$
$= 15.2$

(c) In which class interval does the upper quartile lie? [1]

Time	0 - 20	20 - 30	30 - 40	40 - 60	60 - 90
Cumulative freq.	440	1160	2080	2380	2500

$Q_3: \frac{3 \times 2500}{4} = 1875$ 1875 (Q_3)

Q_3 lies in $30 \leq t < 40$

It was later discovered that the times taken to travel to college by two students were incorrectly recorded. One student's time was recorded as 15 instead of 5 and the other's time was recorded as 65 instead of 75.

(d) Without doing any further calculations, state with a reason whether the estimate of the standard deviation in part **(b)** would be increased, decreased or stay the same. [1]

It would stay the same: 15 and 5 would both be in the $0 \leq t < 20$ class, 65 and 75 would both be in $60 \leq t < 90$, so it doesn't affect the calculations in part (b).

4 Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

1H: F₁ F₂ F₃ B

H T T T

T H T T

T T H T

$\left.\right\} \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{3}{10} = \frac{3}{80}$

T T T H $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{7}{10} = \frac{7}{80}$

$P(1H) = 3\times\frac{3}{80} + \frac{7}{80}$

$= \frac{16}{80} = \frac{1}{5}$ QED

2H: F₁ F₂ F₃ B

H H T T

H T H T

T H H T

$\left.\right\} \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{3}{10} = \frac{3}{80}$

T H T H

H T T H

T T H H

$\left.\right\} \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{7}{10} = \frac{7}{80}$

$P(2H) = 3\times\frac{3}{80} + 3\times\frac{7}{80}$

$= \frac{30}{80} = \frac{3}{8}$ $\left(b = \frac{3}{8}\right)$

$P(3H) = 1 - \left(\frac{3}{80} + \frac{1}{5} + \frac{3}{8} + \frac{7}{80}\right)$

$= 1 - \frac{7}{10}$

$= \frac{3}{10}$ $\left(c = \frac{3}{10}\right)$

(b) Find $E(X)$. [1]

$E(X) = 0\times\frac{3}{80} + 1\times\frac{1}{5} + 2\times\frac{3}{8} + 3\times\frac{3}{10} + 4\times\frac{7}{80}$

$= 0 + \frac{1}{5} + \frac{6}{8} + \frac{9}{10} + \frac{28}{80}$

$= \frac{11}{5}$

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

$X \sim B(10, \frac{1}{5})$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{10}C_0 \times (\frac{1}{5})^0 \times (\frac{4}{5})^{10} + {}^{10}C_1 \times (\frac{1}{5})^1 \times (\frac{4}{5})^9 + {}^{10}C_2 \times (\frac{1}{5})^2 \times (\frac{4}{5})^8$

$= \underline{0.678}$

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

$X \sim Geo(\frac{1}{5})$

$P(X=7) + P(X=8) = (\frac{1}{5}) \times (\frac{4}{5})^6 + (\frac{1}{5}) \times (\frac{4}{5})^7$

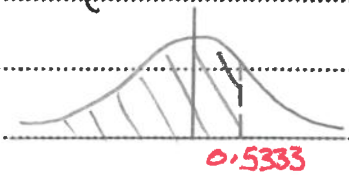
$= \underline{0.0944}$

5 The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2,\ 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

$P(L < 6) = P\left(Z < \frac{6 - 5.2}{1.5}\right)$

$= P(Z < 0.5333)$



$= \Phi(0.533)$

$= \underline{0.7029}$

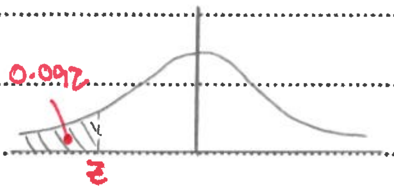
The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

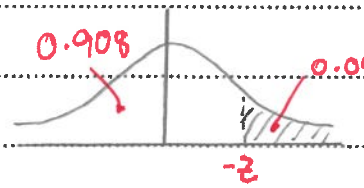
(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

$P(X < 3) = \frac{46}{500}$

$P(X < 3) = 0.092$

$P\left(Z < \frac{3 - \mu}{\sigma}\right) = 0.092$





$0.908 = \Phi(1.329)$

$\therefore z = -1.329$

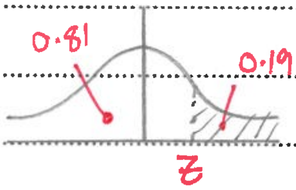
$$\frac{3-\mu}{\sigma} = -1.329$$

$3-\mu = -1.329\sigma$ ①

$$P(X>8) = \frac{95}{500}$$

$P(X>8) = 0.19$

$$P\left(Z > \frac{8-\mu}{\sigma}\right) = 0.19$$



$0.81 = \Phi(0.878)$

$\therefore\ z = 0.878$

$$\frac{8-\mu}{\sigma} = 0.878$$

$8-\mu = 0.878\sigma$ ②

②-①: $5 = 2.207\sigma$

$\sigma = \underline{2.27}$ ← store

→①: $3-\mu = -1.329 \times 2.265...$

$\mu = 3 + 3.01087...$

$\mu = \underline{6.01}$

(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

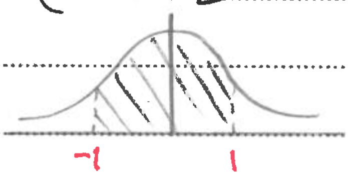
$6.01 + 2.27 = 8.276$ (use unrounded values)

$6.01 - 2.27 = 3.745$

Within 1sd: $P(3.745 < X < 8.276)$

$$P\left(\frac{3.745-6.01}{2.27} < Z < \frac{8.276-6.01}{2.27}\right)$$

Within 1s.d.: $P(-1 < Z < 1)$ ← can start straight from here



$P(Z<1) - P(Z<-1)$

$= \Phi(1) - (1-\Phi(1))$

$= 0.8413 - (1-0.8413)$

$= 0.6826$

Outside 1s.d. $= 1-0.6826$

$= \underline{0.3174}$

number $= 0.3174 \times 2000$

$= 634.8$

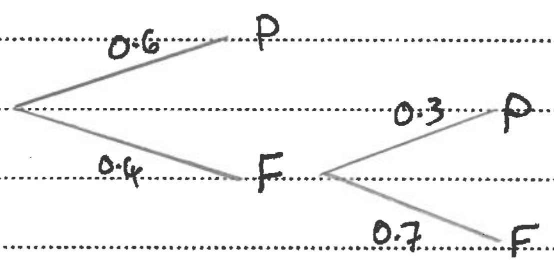
$\simeq \underline{635 \text{ leaves}}$

6 Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

Level 1:

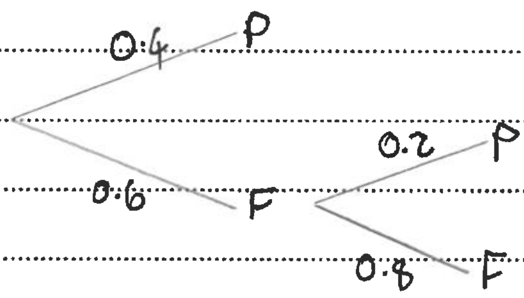


$P(P_1) + P(F_1P_1) = 0.6 + 0.4\times0.3$

$= \underline{0.72}$

(b) Find the probability that Janice finishes the game. [3]

Level 2:



$P(P_2) + P(F_2P_2) = 0.4 + 0.6\times0.2$

$= 0.52$

Probability she finishes = probability of passing both levels

$= 0.72 \times 0.52$

$= \underline{0.3744}$

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

$P(X)$ $P(Y)$

$P(X \cap Y) = P(X|Y) \times P(Y)$

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$P(X \cap Y) = P(P_1F_2P_2) + P(F_1P_1P_2)$

$= 0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.4$

$= 0.072 + 0.048$

$= 0.12$

$P(X|Y) = \frac{0.12}{0.3744}$

$= 0.321$

