1 An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. [1]

$P(5) = \frac{1}{6}$ $\mu = \frac{1}{p}$

$= \frac{1}{1/6} = 6$

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

$P(3 < X < 8) = P(4 \leqslant X \leqslant 7)$

$= P(X \leqslant 7) - P(X \leqslant 3)$

$= (1 - q^7) - (1 - q^3)$

7 failures 3 failures

$= \left(1 - \left(\frac{5}{6}\right)^7\right) - \left(1 - \left(\frac{5}{6}\right)^3\right)$

$= 0.300$

(c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

$P(X < 10) = P(X \leqslant 9)$

$= 1 - q^9$

probability of 9 failures

$= 1 - \left(\frac{5}{6}\right)^9$

$= 0.806$

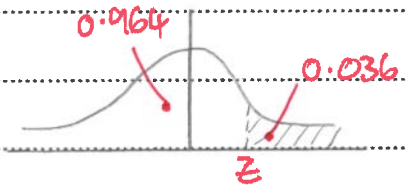
2 The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

$P(W > 1.10) = \frac{72}{2000}$

$P(W > 1.10) = 0.036$

$P\left(Z > \frac{1.10 - 1.04}{\sigma}\right) = 0.036$



$0.964 = \Phi(1.798)$

$\therefore z = 1.798$

$\frac{1.10 - 1.04}{\sigma} = 1.798$

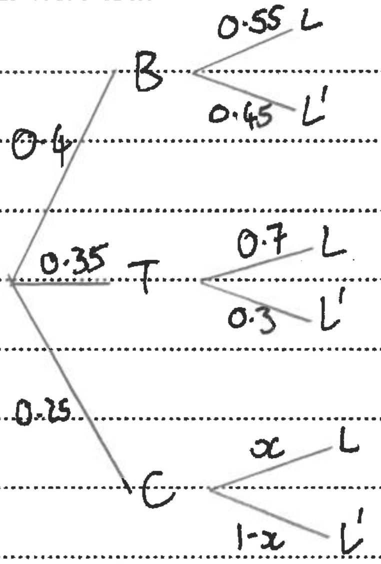
$0.06 = 1.798\sigma$

$\sigma = \underline{0.0334}$

3 On each day that Alexa goes to work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that she arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the probability that she arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. [3]



$P(\text{not late}) = 0.4 \times 0.45 + 0.35 \times 0.3 + 0.25(1-x)$

$= 0.18 + 0.105 + 0.25 - 0.25x$

$0.48 = 0.535 - 0.25x$

$0.25x = 0.055$

$x = 0.22$

(b) Find the probability that Alexa travels to work by train given that she arrives late. [3]

$P(T \cap L) = P(T|L) \times P(L)$

$P(T|L) = \dfrac{P(T \cap L)}{P(L)}$

$P(T \cap L) = 0.35 \times 0.7$

$= 0.245$

$P(L) = 1 - 0.48$

$= 0.52$

$P(T|L) = \dfrac{0.245}{0.52}$

$= 0.471$

4 A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

	1	2	2
−2	−1	0	0
0	1	2	2
1	2	3	3

x	−1	0	1	2	3
$P(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{2}{9}$

(b) Find E(X) and Var(X). [3]

$$E(X) = -1\times\frac{1}{9} + 0\times\frac{2}{9} + 1\times\frac{1}{9} + 2\times\frac{3}{9} + 3\times\frac{2}{9}$$

$$= \frac{-1}{9} + 0 + \frac{1}{9} + \frac{6}{9} + \frac{6}{9}$$

$$= \underline{\frac{4}{3}}$$

$$Var(X) = (-1)^2\times\frac{1}{9} + 0^2\times\frac{2}{9} + 1^2\times\frac{1}{9} + 2^2\times\frac{3}{9} + 3^2\times\frac{2}{9} - \left(\frac{4}{3}\right)^2$$

$$= \frac{1}{9} + 0 + \frac{1}{9} + \frac{12}{9} + \frac{18}{9} - \frac{16}{9}$$

$$= \underline{\frac{16}{9}}$$

5 Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

$P(L') = 0.15 + 0.55$
$= 0.7$
$P(L'L'L') = 0.7 \times 0.7 \times 0.7$
$= 0.343$

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

$F \sim B(9, 0.15)$

$P(F \geqslant 3) = 1 - (P(0) + P(1) + P(2))$
$= 1 - ({}^9C_0 \times 0.15^0 \times 0.85^9 + {}^9C_1 \times 0.15^1 \times 0.85^8 + {}^9C_2 \times 0.15^2 \times 0.85^7)$
$= 0.141$

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

$F \sim B(60, 0.15)$

$\mu = 60 \times 0.15$ $\qquad \sigma^2 = 9(0.85)$

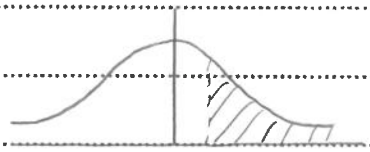
$= 9$ $\qquad = 7.65$

$F \sim N(9, 7.65)$

$P(F \geqslant 12) \rightarrow P(F > 11.5)$ (continuity correction)

$P\left(Z > \frac{11.5 - 9}{\sqrt{7.65}}\right)$

$= P(Z > 0.9039)$



0.9039

$= 1 - \Phi(0.904)$

$= 1 - 0.8169$

$= 0.1831$

6 (a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

TOOOMRRW

$$\frac{8!}{2! \times 3!} = \underline{3360}$$

2Rs 3Os

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

R _ _ _ _ _ _ R

With three Os together:

R _ _ _ _ R

3 Os are one object → OOO T M W

4 objects arranged in 4 spaces: 4!

With no restrictions:

R _ _ _ _ _ _ R

O O O T M W

$$\frac{6!}{3!}$$

3Os

ANS: $\frac{6!}{3!} - 4! = \underline{96}$

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

O R _ _ $\quad {}^3C_1 \times {}^2C_1 \times {}^3C_2 = 18$

1 O from 3 $\quad$ 1 R from 2 $\quad$ 2 letters from T, M, W

O O R _ $\quad {}^3C_2 \times {}^2C_1 \times {}^3C_1 = 18$

O O O R $\quad {}^3C_3 \times {}^2C_1 = 2$

O R R _ $\quad {}^3C_1 \times {}^2C_2 \times {}^3C_1 = 9$

O O R R $\quad {}^3C_2 \times {}^2C_2 = 3$

$18 + 18 + 2 + 9 + 3 = 50$

Number of selections without restrictions $= {}^8C_4$

$= 70$

Probability $= \frac{50}{70}$

$= \frac{5}{7}$

7 The heights, in cm, of the 11 basketball players in each of two clubs, the Amazons and the Giants, are shown below.

Amazons	205	198	181	182	190	215	201	178	202	196	184
Giants	175	182	184	187	189	192	193	195	195	195	204

(a) State an advantage of using a stem-and-leaf diagram compared to a box-and-whisker plot to illustrate this information. [1]

A stem and leaf diagram shows the values of all of the data.

(b) Represent the data by drawing a back-to-back stem-and-leaf diagram with Amazons on the left-hand side of the diagram. [4]

Amazons				Giants				
		8	17	5				
4	$\overset{Q_1}{2}$	1	18	2	4	7	9	
8	$\overset{Q_2}{6}$	0	19	2	3	5	5	5
5	$\overset{Q_3}{2}$	1	20	4				
		5	21					

Key: 8|17|5 means 178cm for Amazons and 175cm for Giants

(c) Find the interquartile range of the heights of the players in the Amazons. [2]

Q_1: $\frac{11+1}{4} = 3^{rd}$ Q_2: $\frac{11+1}{2} = 6^{th}$ Q_3: $\frac{3(11+1)}{4} = 9^{th}$

$Q_1 = 182$ $Q_2 = 196$ $Q_3 = 202$

$IQR = 202 - 182$
$= 20cm$

Four new players join the Amazons. The mean height of the 15 players in the Amazons is now 191.2 cm. The heights of three of the new players are 180 cm, 185 cm and 190 cm.

(d) Find the height of the fourth new player. [3]

Total height of all 15 players $= 191.2 \times 15$
$= 2868$

Sum of all other 14 players' heights $= 2687$ add up data

Height of missing player $= 2868 - 2687$
$= 181cm$

