1 A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

4 Red, 1 Blue:

$$ {}^{8}C_4 \times {}^{4}C_1 = 280 $$

pick 4 reds from 8.

pick 1 blue from 4

5 Red, no Blue:

$$ {}^{8}C_5 = 56 $$

4 Blue, 1 Red:

$$ {}^{4}C_4 \times {}^{8}C_1 = 8 $$

5 Blue not possible.

$$ 280 + 56 + 8 = \underline{\underline{344}} $$

2 A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

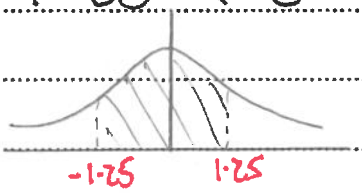
$25.2 + 0.5 = 25.7$

$25.2 - 0.5 = 24.7$

$P(24.7 < L < 25.7)$

$P\left(\frac{24.7 - 25.2}{0.4} < Z < \frac{25.7 - 25.2}{0.4}\right)$

$P(-1.25 < Z < 1.25)$



$= \Phi(1.25) - (1 - \Phi(1.25))$

$= 0.8944 - (1 - 0.8944)$

$= 0.7888$

Number $= 0.7888 \times 500$

$= 394.4$

≈ 394 rods

3 **(a)** How many different arrangements are there of the 8 letters in the word RELEASED? [1]

REEELASD

$$\frac{8!}{3!} = \underline{6720}$$

3 Es

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

LED

one object

$$\frac{6!}{2!} = \underline{360}$$

remaining 2 Es

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

A and D together:

AD

one object

$$\frac{7!}{3!} \times 2! = 1680$$

3 Es

could be AD or DA

Without restrictions:

$$\frac{8!}{3!} = 6720$$

$\rightarrow 6720 - 1680 = 5040$

$$\text{probability} = \frac{5040}{6720}$$

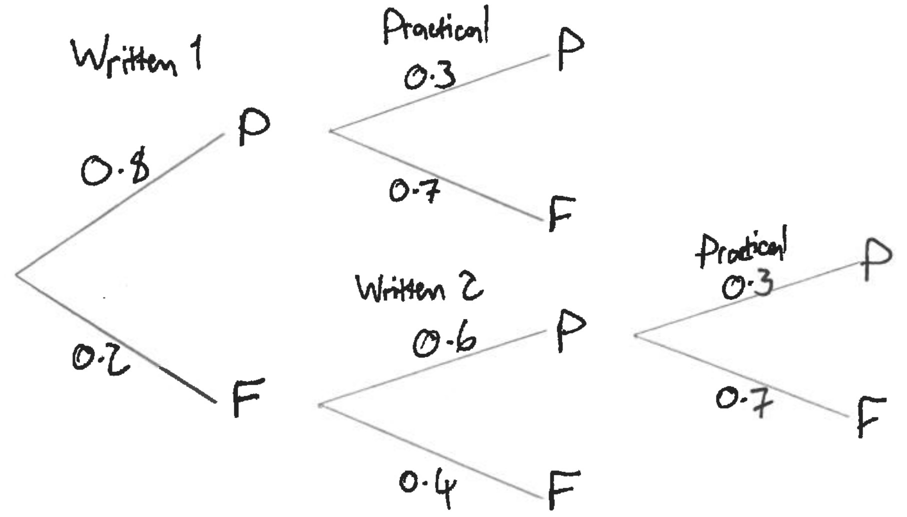
$$= \frac{3}{4}$$

4 To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If a student fails both attempts at the written test, then they cannot attempt the practical test.

The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. [3]



(b) Find the probability that a randomly chosen student will succeed in gaining a place at the college. [2]

P(PP) + P(FPP)

$= 0.8 \times 0.3 + 0.2 \times 0.6 \times 0.3$

$= 0.24 + 0.036$

$= 0.276$

(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [2]

$P(X)$ $P(Y)$

$P(X \cap Y) = P(X|Y) \times P(Y)$

$P(X|Y) = \dfrac{P(X \cap Y)}{P(Y)}$

$P(X \cap Y) = P(PP)$
$= 0.8 \times 0.3$
$= 0.24$

$P(Y) = 0.276$

$P(X|Y) = \dfrac{0.24}{0.276}$

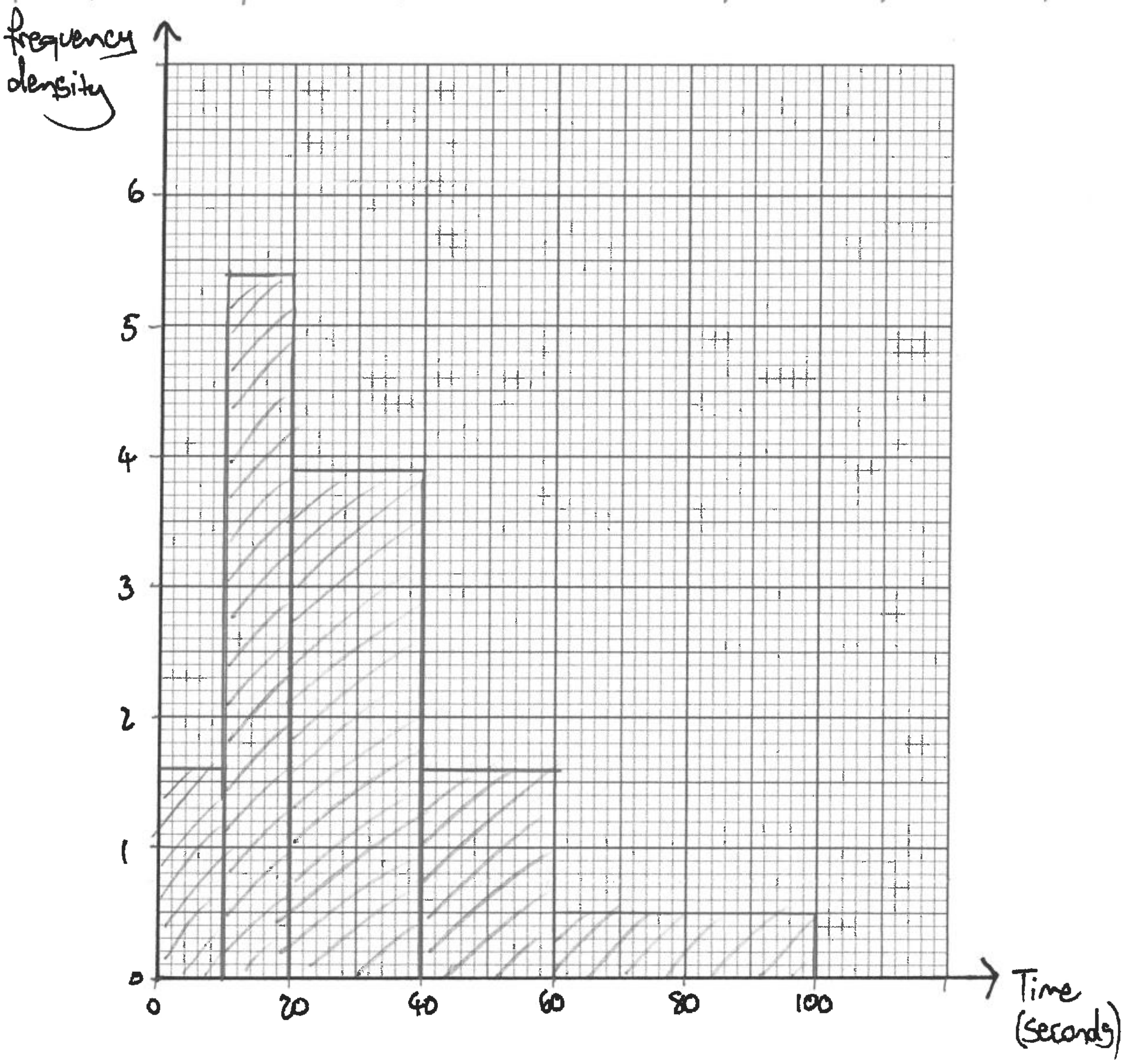
$= \dfrac{20}{23}$

5 The times taken by 200 players to solve a computer puzzle are summarised in the following table.

Time (t seconds)	$0 \leqslant t < 10$	$10 \leqslant t < 20$	$20 \leqslant t < 40$	$40 \leqslant t < 60$	$60 \leqslant t < 100$
Number of players	16	54	78	32	20

(a) Draw a histogram to represent this information. [4]

class width	10	10	20	20	40
f.d.	1.6	5.4	3.9	1.6	0.5



(b) Calculate an estimate of the mean time taken by these 200 players. [2]

Mid-point (t)	Frequency (f)	$f \times t$
5	16	80
15	54	810
30	78	2340
50	32	1600
80	20	1600
	$\Sigma f = 200$	$\Sigma ft = 6430$

$$\bar{t} = \frac{6430}{200}$$

$$= 32.15$$

(c) Find the greatest possible value of the interquartile range of these times. [2]

Time	0 - 10	10 - 20	20 - 40	40 - 60	60 - 100
cumulative freq	16	70	148	180	200

50 Q_1 150 Q_3

Q_1: $\frac{200}{4} = 50$

Q_3: $\frac{3 \times 200}{4} = 150$

Greatest possible IQR = biggest possible Q_3 − smallest possible Q_1

$= 60 - 10$

$= 50$

6 In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

$C \sim B(12, 0.6)$

$P(C<10) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.6^{10} \times 0.4^{2} + {}^{12}C_{11} \times 0.6^{11} \times 0.4^{1} + {}^{12}C_{12} \times 0.6^{12} \times 0.4^{0})$

$= \underline{\underline{0.917}}$

(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

$C \sim B(150, 0.6)$

$\mu = 150 \times 0.6 = 90$

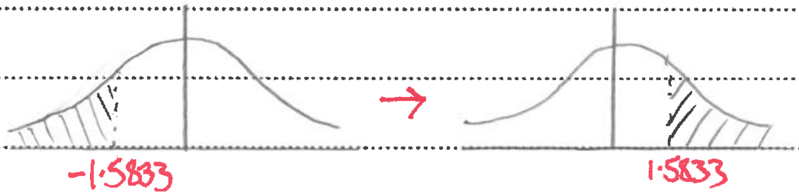
$\sigma^2 = 90(0.4) = 36$

$C \sim N(90, 36)$

$P(C < 81) \rightarrow P(C < 80.5)$

$P\left(Z < \frac{80.5 - 90}{\sqrt{36}}\right)$

$= P(Z < -1.5833)$



$= 1 - \Phi(1.583)$

$= 1 - 0.9433$

$= \underline{0.0567}$

(c) Justify the use of your approximation in part (b). [1]

$np = 150 \times 0.6$
$= 90$

$nq = 150 \times 0.4$
$= 60$

Both are greater than 5 so we can use the Normal approximation.

7 Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X = 3) = \frac{6}{35}$. [2]

3 tins of carrots, 4 other tins:

$$\frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} = \frac{6}{35} \quad \text{QED}$$

not carrot   not carrot   carrot

(b) Draw up the probability distribution table for X. [4]

$P(X=1) = \frac{3}{7} \left(= \frac{15}{35}\right)$

$P(X=2) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7} \left(= \frac{10}{35}\right)$

$P(X=3) = \frac{6}{35}$

$P(X=4) = \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{35}$

$P(X=5) = \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{1}{35}$

x	1	2	3	4	5
$P(X=x)$	$15/35$	$10/35$	$6/35$	$3/35$	$1/35$

(c) Find Var(X). [3]

$E(X) = 1 \times \frac{15}{35} + 2 \times \frac{10}{35} + 3 \times \frac{6}{35} + 4 \times \frac{3}{35} + 5 \times \frac{1}{35}$

$= \frac{15}{35} + \frac{20}{35} + \frac{18}{35} + \frac{12}{35} + \frac{5}{35}$

$= \frac{70}{35} = \underline{2}$

$Var(X) = 1^2 \times \frac{15}{35} + 2^2 \times \frac{10}{35} + 3^2 \times \frac{6}{35} + 4^2 \times \frac{3}{35} + 5^2 \times \frac{1}{35} - (E(X))^2$

$= \frac{15}{35} + \frac{40}{35} + \frac{54}{35} + \frac{48}{35} + \frac{25}{35} - (2)^2$

$= \frac{182}{35} - 4$

$= \underline{\underline{\frac{6}{5}}}$