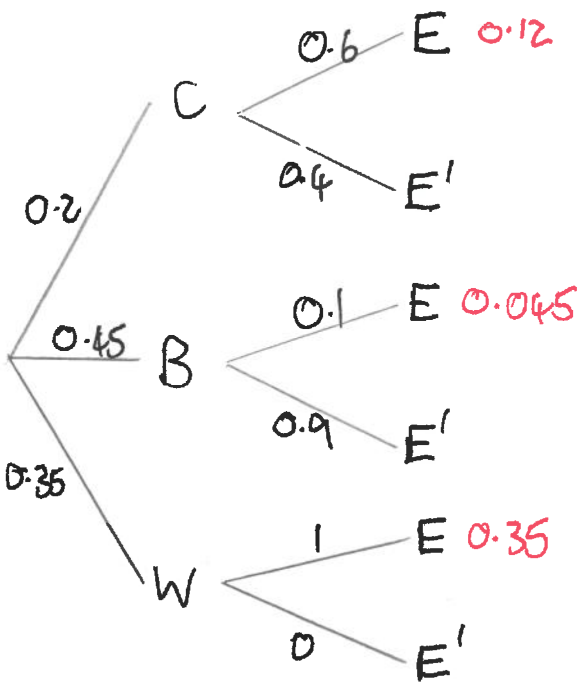
1 Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]



(b) Find the probability that Juan goes to college by car given that he arrives early. [4]

$P(C \cap E) = P(C|E) \times P(E)$

$P(C|E) = \dfrac{P(C \cap E)}{P(E)}$

$P(C \cap E) = 0.12$

$P(E) = 0.12 + 0.045 + 0.35$

$= 0.515$

$P(C|E) = \dfrac{0.12}{0.515} = \dfrac{24}{103}$

N.B: Error in mark scheme: $\frac{12}{515}$ is incorrect, it should be $\frac{120}{515}$.

2 In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car. [1]

$0.22 \times 0.22 \times 0.22 = \underline{0.010648}$

(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4. [3]

$C \sim B(16, 0.22)$

$P(2 \leqslant C \leqslant 4) = P(2) + P(3) + P(4)$

$= {}^{16}C_2 \times 0.22^2 \times 0.78^{14} + {}^{16}C_3 \times 0.22^3 \times 0.78^{13}$
$+ {}^{16}C_4 \times 0.22^4 \times 0.78^{12}$

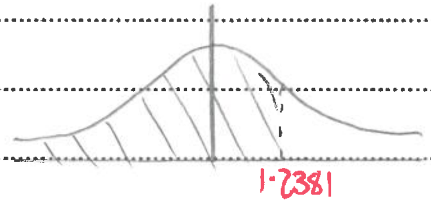
$= \underline{0.631}$

3 In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

$$P(X < 21) = P\left(Z < \frac{21 - 15.8}{4.2}\right)$$

$$= P(Z < 1.2381)$$



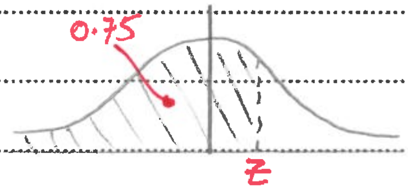
$$= \Phi(1.238)$$

$$= \underline{0.8922}$$

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

$$P(X < k) = 0.75$$

$$P\left(Z < \frac{k - 15.8}{4.2}\right) = 0.75$$



$$\underline{0.75} = \Phi(0.674)$$

critical value

$$\therefore z = 0.674$$

$$\frac{k - 15.8}{4.2} = 0.674$$

$$k - 15.8 = 2.8308$$

$$k = \underline{18.6}\text{ hrs}$$

4 A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

	1	2	2	3
-2	-1	0	0	1
-1	0	1	1	2
1	2	3	3	4

x	-1	0	1	2	3	4
$P(X=x)$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

(b) Find Var(X). [3]

$$Var(X) = (-1)^2 \times \frac{1}{12} + 0^2 \times \frac{3}{12} + 1^2 \times \frac{3}{12} + 2^2 \times \frac{2}{12} + 3^2 \times \frac{2}{12} + 4^2 \times \frac{1}{12} - (E(X))^2$$

$$= \frac{1}{12} + 0 + \frac{3}{12} + \frac{8}{12} + \frac{18}{12} + \frac{16}{12} - (E(X))^2$$

$$= \frac{46}{12} - (E(X))^2$$

$$E(X) = -1 \times \frac{1}{12} + 0 \times \frac{3}{12} + 1 \times \frac{3}{12} + 2 \times \frac{2}{12} + 3 \times \frac{2}{12} + 4 \times \frac{1}{12}$$

$$= \frac{-1}{12} + 0 + \frac{3}{12} + \frac{4}{12} + \frac{6}{12} + \frac{4}{12}$$

$$= \frac{16}{12}$$

$$Var(X) = \frac{46}{12} - \left(\frac{16}{12}\right)^2 = \frac{37}{18}$$

5 A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

$X \sim Geo\left(\frac{1}{4}\right)$

$P(T \cap T) = \frac{1}{2} \times \frac{1}{2}$

$\mu = \frac{1}{\frac{1}{4}}$

$= \underline{4}$

(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

$P(X = 3) = \frac{1}{4} \times \left(\frac{3}{4}\right)^2$

$= \underline{\frac{9}{64}}$

(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

$P(X < 6) = P(X \leqslant 5)$

$= 1 - q^5$

probability of 5 failures

$= 1 - \left(\frac{3}{4}\right)^5$

$= \underline{0.763}$

On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

$X \sim B(80, \frac{1}{4})$

$\mu = 80 \times \frac{1}{4}$ $\sigma^2 = 20(\frac{3}{4})$

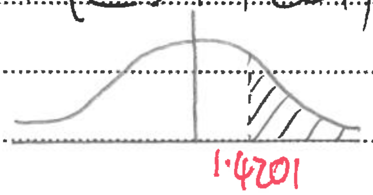
$= 20$ $= 15$

$X \sim N(20, 15)$

$P(X > 25) \rightarrow P(X > 25.5)$ (continuity correction)

$P(Z > \frac{25.5 - 20}{\sqrt{15}})$

$= P(Z > 1.4201)$



$= 1 - \Phi(1.420)$

$= 1 - 0.9222$

$= 0.0778$

6 The annual salaries, in thousands of dollars, for 11 employees at each of two companies A and B are shown below.

Company A	30	32	35	41	41	42	47	49	52	53	64
Company B	26	47	30	52	41	38	35	42	49	31	42

(a) Represent the data by drawing a back-to-back stem-and-leaf diagram with company A on the left-hand side of the diagram. [4]

Company A		Company B
	2	6
Q_1 5 2 0	3	0 1 5 8
9 7 2 Q_2 1 1	4	1 2 2 7 9
3 Q_3 2	5	2
4	6	

Key: 1|4|2 means \$41000 for company A and \$42000 for company B

(b) Find the median and the interquartile range of the salaries of the employees in company A. [3]

$Q_1: \frac{11+1}{4} = 3^{rd}$ $Q_2: \frac{11+1}{2} = 6^{th}$ $Q_3: \frac{3(11+1)}{4} = 9^{th}$

$Q_1 = 35000$ $Q_2 = 42000$ $Q_3 = 52000$

Median = $\$42000$ IQR = $52000 - 35000$ $= \$17000$

A new employee joins company B. The mean salary of the 12 employees is now \$38 500.

(c) Find the salary of the new employee. [3]

Sum of salaries of 11 employees = 433 000

add up data in diagram

Sum of salaries of all 12 employees = 12×38500 $= 462000$

$462000 - 433000 = \$29000$

7 (a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

CEELLSTIA

$$\frac{9!}{2! \times 2!} = \underline{90\,720}$$

2Es 2Ls

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

C _ _ _ T _ _ _ E

fixed fixed fixed

$$\frac{6!}{2!} = \underline{360}$$

2Ls

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

Two Es together:

_ _ _ _ _ _ _ _

one object → EE

$$\frac{8!}{2!} = 20\,160$$

2Ls

No restrictions: 90 720 (from part (a))

Es not together: 90720 − 20160 = 70560

probability $= \frac{70560}{90\,720} = \underline{\frac{7}{9}}$

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

1 E, 0 L: E _ _ _ _ $^5C_4 = 5$

pick 4 from CSTIA

2 E, 0 L: E E _ _ _ $^5C_3 = 10$

1 E, 1 L: E L _ _ _ $^5C_3 = 10$

2 E, 1 L: E E L _ _ $^5C_2 = 10$

$5 + 10 + 10 + 10 = \underline{\underline{35}}$