1 For n values of the variable x, it is given that

$$\Sigma(x - 50) = 144 \quad \text{and} \quad \Sigma x = 944. \ (1)$$

Find the value of n. [3]

$$\Sigma(x - 50) = 144$$
$$\Sigma x = 144 + 50n \ (2)$$

$$(1) = (2)$$
$$144 + 50n = 944$$
$$50n = 800$$
$$\underline{\underline{n = 16}}$$

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2 A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

$\frac{56}{500} = \frac{14}{125}$

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

P(S) P(D)

$P(D \cap S) = P(D|S) \times P(S)$

$P(D|S) = \frac{P(D \cap S)}{P(S)}$

$P(D \cap S) = \frac{120}{500}$

$P(S) = \frac{280}{500}$

$P(D|S) = \frac{120/500}{280/500}$

$= \frac{3}{7}$

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

If independent, $P(H) \times P(A \text{ or } B) = P(H \cap A \text{ or } B)$

$P(H) = \frac{220}{500}$

$P(A \text{ or } B) = \frac{86}{500} + \frac{156}{500}$

$= \frac{242}{500}$

$P(H \cap A \text{ or } B) = \frac{32}{500} + \frac{72}{500}$

$= \frac{104}{500}$

$= \frac{26}{125}$

$P(H) \times P(A \text{ or } B) = \frac{220}{500} \times \frac{242}{500}$

$= \frac{1331}{6250}$

$\frac{1331}{6250} \neq \frac{26}{125}$ so ~~not~~ not independent.

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3 Two machines, A and B, produce metal rods of a certain type. The lengths, in metres, of 19 rods produced by machine A and 19 rods produced by machine B are shown in the following back-to-back stem-and-leaf diagram.

	A		B
		21	1 2 4
	7 6 3 0	22	2 4 5 5 6
Q2	8 7 4 3 1 1 Q1	23	0 2 6 8 9 9
Q3	5 5 5 3 2	24	3 3 4 6
	4 3 1 0	25	6

Key: 7 | 22 | 4 means 0.227 m for machine A and 0.224 m for machine B.

(a) Find the median and the interquartile range for machine A. [3]

$Q_1: \frac{19+1}{4} = 5^{th}$ $\quad Q_2: \frac{19+1}{2} = 10^{th}$ $\quad Q_3: \frac{3(19+1)}{4} = 15^{th}$

$Q_1 = 0.231$ $\quad Q_2 = 0.238$ $\quad Q_3 = 0.245$

Median $= 0.238$ m $\quad$ IQR $= 0.245 - 0.231$

$= 0.014$ m

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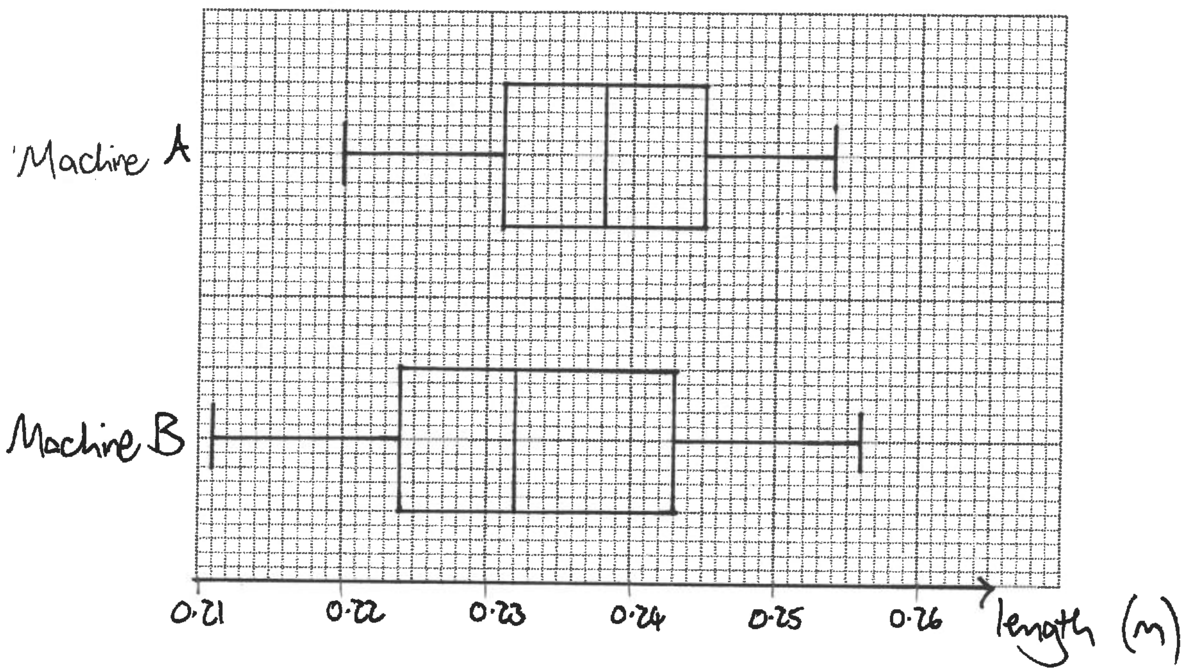
It is given that for machine B the median is 0.232 m, the lower quartile is 0.224 m and the upper quartile is 0.243 m.

(b) Draw box-and-whisker plots for A and B. [3]



(c) Hence make two comparisons between the lengths of the rods produced by machine A and those produced by machine B. [2]

On average, Machine A produces longer rods than machine B.

Machine A produces rods of a more consistent length than Machine B.

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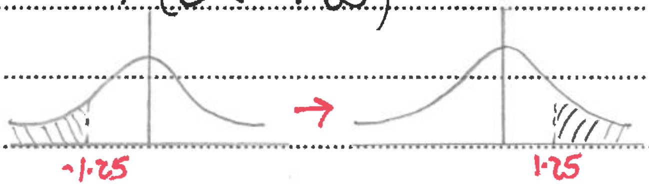
4 Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. [3]

$P(L < 25) = P\left(Z < \frac{25-40}{12}\right)$

$= P(Z < -1.25)$



$= 1 - \Phi(1.25)$

$= 1 - 0.8944$

$= \underline{0.1056}$

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places. [2]

$1 - 0.1056 = 0.8944$

$0.8944 \div 3 = \underline{0.298}$ (3dp) QED

↑ store full answer

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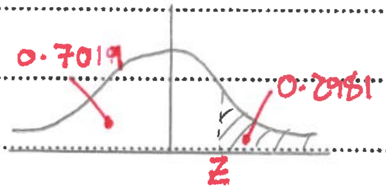
(c) Find the height above which trees are classified as tall. [3]

$P(L > l) = 0.2981...$

$P\left(Z > \dfrac{l-40}{12}\right) = 0.2981...$



$0.7019 = \Phi(0.53)$

$\therefore z = 0.53$

$\dfrac{l-40}{12} = 0.53$

$l - 40 = 6.36$

$l = 46.36$

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5 A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

	1	2	3
1	1	2	3
1	1	2	3
2	2	2	3
2	2	2	3
3	3	3	3

$P(X=3) = \frac{7}{15}$

(b) Draw up the probability distribution table for X. [3]

x	1	2	3
$P(X=x)$	$^{2}/_{15}$	$\frac{6}{15}$	$\frac{7}{15}$

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(c) Find E(X) and Var(X). [3]

$E(X) = 1 \times \frac{2}{15} + 2 \times \frac{6}{15} + 3 \times \frac{7}{15}$

$= \frac{2}{15} + \frac{12}{15} + \frac{21}{15}$

$= \frac{35}{15} = \frac{7}{3}$

$Var(X) = 1^2 \times \frac{2}{15} + 2^2 \times \frac{6}{15} + 3^2 \times \frac{7}{15} - (E(x))^2$

$= \frac{2}{15} + \frac{24}{15} + \frac{63}{15} - \left(\frac{7}{3}\right)^2$

$= \frac{89}{15} - \frac{49}{9}$

$= \frac{22}{45}$

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6 (a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

SUMMMEERTI

E _ _ _ _ _ _ _ _ E

fixed fixed

$\frac{8!}{3!} = 6720$

3 Ms

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

Es together:

_ _ _ _ _ _ _ _ _

One object → EE

$\frac{9!}{3!} = 60\,480$

3 Ms

No restrictions:

$\frac{10!}{3! \times 2!} = 302\,400$

3 Ms 2 Es

$302\,400 - 60\,480 = 241\,920$

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(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

1M, 1E: M E _ _ $^5C_2 = 10$

pick 2 from SURTI

2M, 1E: M M E _ $^5C_1 = 5$

3M, 1E: M M M E $^5C_0 = 1$

$10 + 5 + 1 = \underline{16}$

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7 On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

$M \sim B(12, 0.72)$

$P(M \leq 9) = 1 - (P(10) + P(11) + P(12))$

$= 1 - ({}^{12}C_{10} \times 0.72^{10} \times 0.28^2 + {}^{12}C_{11} \times 0.72^{11} \times 0.28^1 + {}^{12}C_{12} \times 0.72^{12} \times 0.28^0)$

$= \underline{0.696}$

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

$M \sim Geo(0.72)$

= Probability that first success is on fourth trial:

2nd Jan 3rd Jan 4th Jan 5th Jan

X X X ✓

$= 0.72 \times 0.28^3$

$= \underline{0.0158}$

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(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

$$M \sim B(100, 0.72)$$

$\mu = 100 \times 0.72$ $\qquad\qquad$ $\sigma^2 = 72(0.28)$

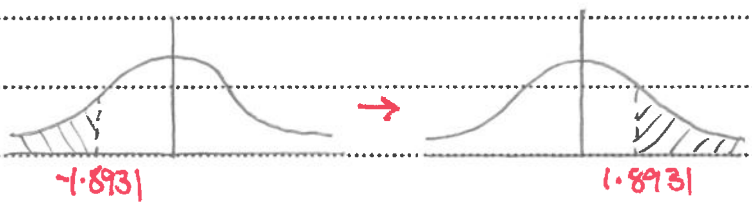
$= 72$ $\qquad\qquad\qquad\qquad$ $= 20.16$

$$M \sim N(72, 20.16)$$

$P(M < 64) \rightarrow P(M < 63.5)$ (continuity correction)

$$P\left(Z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$$

$$= P(Z < -1.8931)$$



$$= 1 - \Phi(1.893)$$

$$= 1 - 0.9708$$

$$= 0.0292$$

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