1 The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

$$P(\text{score of } 4) = P(1,3) + P(3,1) + P(2,2)$$
$$= \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6}$$
$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$$
$$= \frac{1}{12}$$

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the mean of X. [1]

$$X \sim \text{Geo}\left(\frac{1}{12}\right)$$
$$\mu = \frac{1}{p} = \frac{1}{1/12}$$
$$= 12$$

(c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

$$P(X=6) = \frac{1}{12} \times \left(\frac{11}{12}\right)^5$$
$$= 0.0539$$

(d) Find $P(X < 8)$. [2]

$$P(X<8) = P(X \leqslant 7)$$
$$= 1 - q^7$$

probability of 7 failures

$$= 1 - \left(\frac{11}{12}\right)^7$$
$$= 0.456$$

2 (a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

JEEEWLLRY

one object → EEE LL ← one object

$6! = \underline{720}$

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

Ls together:

one object → LL

$\frac{8!}{3!} = 6720$

3 Es → 3!

No restrictions:

$\frac{9!}{3! \times 2!} = 30240$

3 Es → 3!, 2Ls → 2!

$30240 - 6720 = \underline{23520}$

3 A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

$P(0) = \frac{{}^5C_0 \times {}^3C_3}{{}^8C_3} = \frac{1}{56}$

no jellies picked

3 chocolates picked from 3

$P(1) = \frac{{}^5C_1 \times {}^3C_2}{{}^8C_3} = \frac{15}{56}$

1 jelly picked from 5

2 chocolates picked from 3

$P(2) = \frac{{}^5C_2 \times {}^3C_1}{{}^8C_3} = \frac{15}{28} \left(= \frac{30}{56}\right)$

$P(3) = \frac{{}^5C_3 \times {}^3C_0}{{}^8C_3} = \frac{5}{28} \left(= \frac{10}{56}\right)$

x	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

$L \sim B(10, 0.64)$

$P(8) = {}^{10}C_8 \times 0.64^8 \times 0.36^2$

$P(9) = {}^{10}C_9 \times 0.64^9 \times 0.36$

$P(10) = {}^{10}C_{10} \times 0.64^{10}$

$P(L \leqslant 7) = 1 - (P(8) + P(9) + P(10))$

$= \underline{0.759}$

4 In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

2P, 1G, 4V: $^8C_2 \times {^4C_1} \times {^6C_4} = 1680$

2 pianists from 8 · 1 guitarist from 4 · 4 violinists from 6

2P, 2G, 3V: $^8C_2 \times {^4C_2} \times {^6C_3} = 3360$

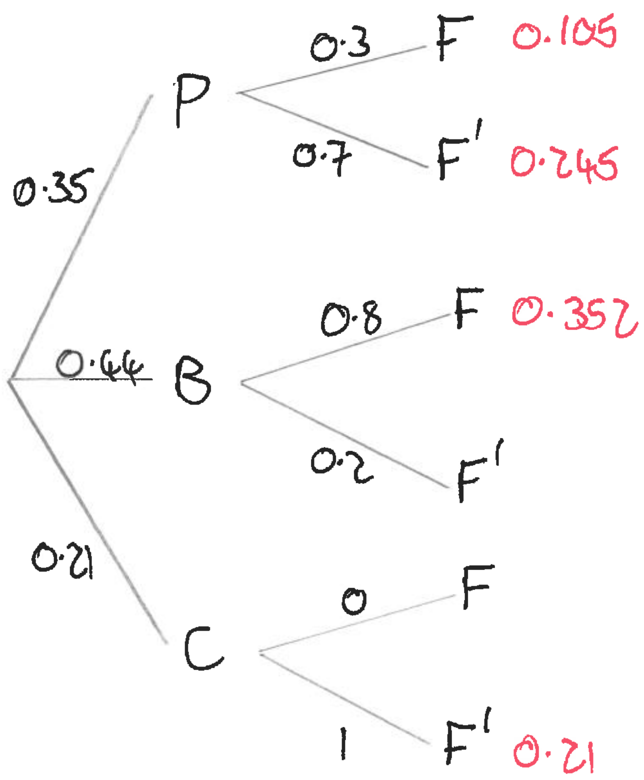
3P, 1G, 3V: $^8C_3 \times {^4C_1} \times {^6C_3} = 4480$

4P, 1G, 2V: $^8C_4 \times {^4C_1} \times {^6C_2} = 4200$

$1680 + 3360 + 4480 + 4200 = \underline{13720}$

5 On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with probabilities 0.35, 0.44, 0.21 respectively. When she cooks a pizza, Rani has some fruit with probability 0.3. When she cooks a burger, she has some fruit with probability 0.8. When she cooks a curry, she never has any fruit.

(a) Draw a fully labelled tree diagram to represent this information. [2]



(b) Find the probability that Rani has some fruit. [2]

$P(F) = 0.105 + 0.352$

$= 0.457$

(c) Find the probability that Rani does not have a burger given that she does not have any fruit. [4]

$P(B' \cap F') = P(B'|F') \times P(F')$

$P(B'|F') = \dfrac{P(B' \cap F')}{P(F')}$

$P(B' \cap F') = P(P \cap F') + P(C \cap F')$

$= 0.245 + 0.21$

$= 0.455$

$P(F') = 1 - P(F)$

$= 1 - 0.457$

$= 0.543$

$P(B'|F') = \dfrac{0.455}{0.543}$

$= \dfrac{455}{543}$

6 The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

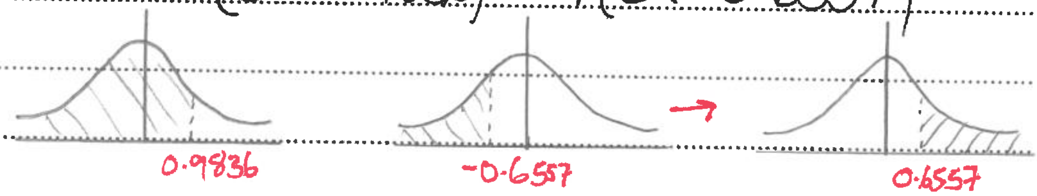
(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

$$P(50 < L < 60)$$

$$= P\left(\frac{50-54}{6.1} < Z < \frac{60-54}{6.1}\right)$$

$$= P(-0.6557 < Z < 0.9836)$$

$$= P(Z < 0.9836) - P(Z < -0.6557)$$



$$= \Phi(0.984) - (1 - \Phi(0.656))$$

$$= 0.8375 - (1 - 0.7441)$$

$$= \underline{0.5816}$$

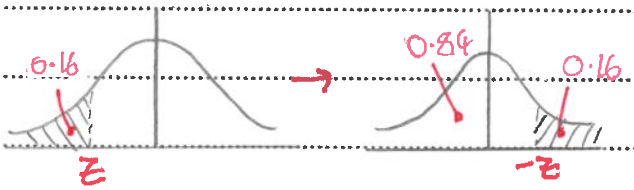
The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

$P(L < 45) = \frac{32}{200}$

$P(L < 45) = 0.16$

$P\left(Z < \frac{45-\mu}{\sigma}\right) = 0.16$



$0.84 = \Phi(0.994)$

$\therefore z = -0.994$

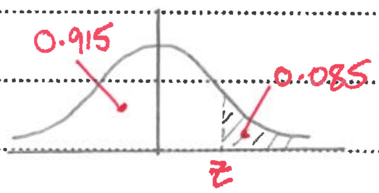
$\frac{45-\mu}{\sigma} = -0.994$

$45 - \mu = -0.994\sigma$ 1

$P(L > 56) = \frac{17}{200}$

$P(L > 56) = 0.085$

$P\left(Z > \frac{56-\mu}{\sigma}\right) = 0.085$



$0.915 = \Phi(1.372)$

$\therefore z = 1.372$

$\frac{56-\mu}{\sigma} = 1.372$

$56 - \mu = 1.372\sigma$ 2

2 − 1:

$11 = 2.366\sigma$

$\sigma = 4.65$ ⇐ store

sub → 2:

$56 - \mu = 1.372 \times 4.65\ldots$

$\mu = 56 - 6.379$

$\mu = 49.6$

7 The numbers of chocolate bars sold per day in a cinema over a period of 100 days are summarised in the following table.

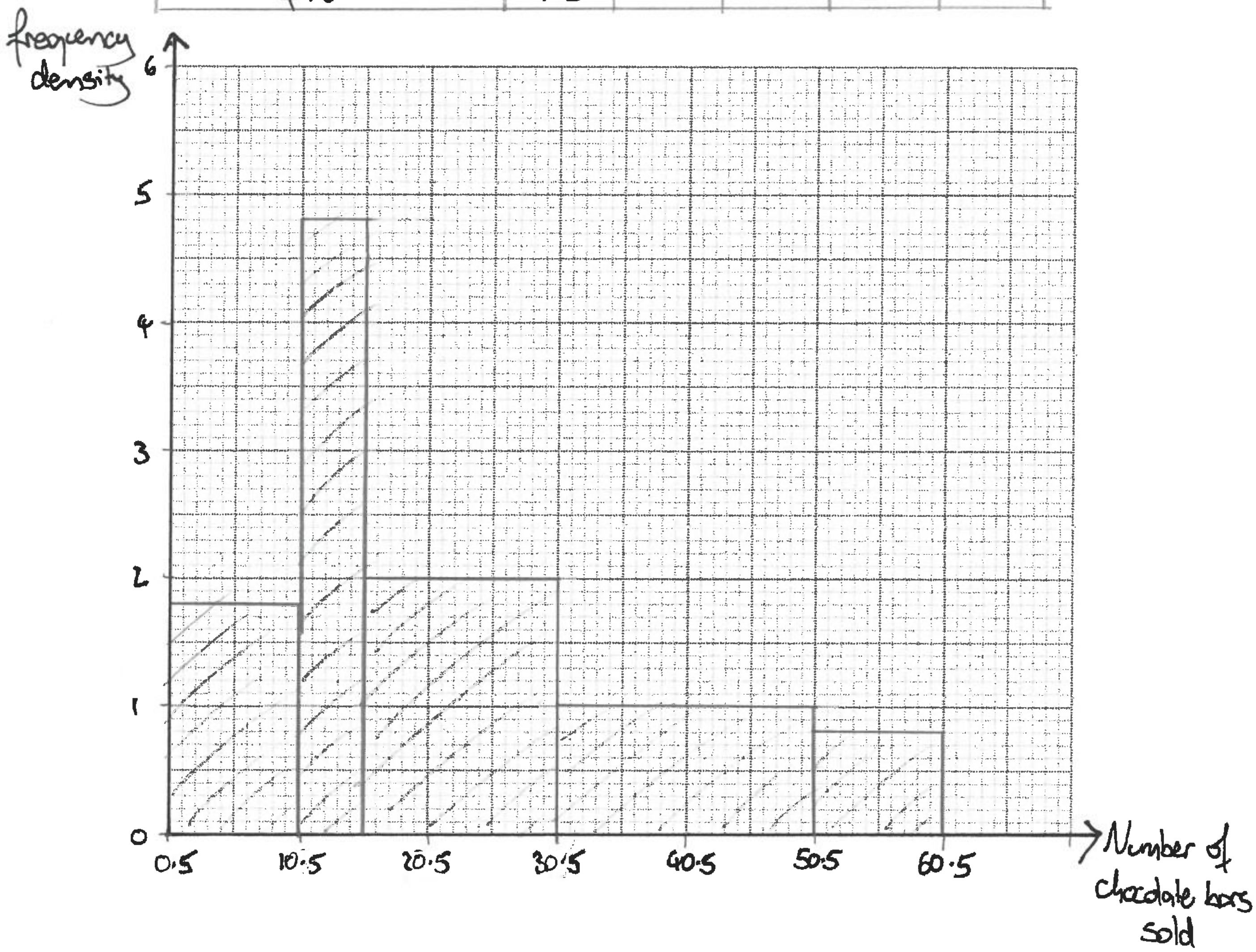
0.5 10.5 15.5 30.5 50.5 60.5

Number of chocolate bars sold	1 – 10	11 – 15	16 – 30	31 – 50	51 – 60
Number of days	18	24	30	20	8

(a) Draw a histogram to represent this information. [5]

10.5 − 0.5

class width	10	5	15	20	10
f.d.	1.8	4.8	2	1	0.8



(b) What is the greatest possible value of the interquartile range for the data? [2]

Number of bars	1-10	11-15	16-30	31-50	51-60
cumulative frequency	18	42	72	92	100

25 Q1   75 Q3

$Q_1: \frac{100}{4} = 25$

$Q_3: \frac{3 \times 100}{4} = 75$ Greatest IQR $= \max(Q_3) - \min(Q_1)$

$= 50 - 11$

$= 39$

(c) Calculate estimates of the mean and standard deviation of the number of chocolate bars sold. [4]

Mid-point (x)	Frequency (f)	$f \times x$
5.5	18	99
13	24	312
23	30	690
40.5	20	810
55.5	8	444
	$\Sigma f = 100$	$\Sigma fx = 2355$

$\bar{x} = \frac{2355}{100}$

$= 23.55$

$\text{Var} = \frac{5.5^2 \times 18 + 13^2 \times 24 + 23^2 \times 30 + 40.5^2 \times 20 + 55.5^2 \times 8}{100} - 23.55^2$

$= 224.57$

$\sigma = \sqrt{224.57}$

$= 15.0$